

Inertia and gravity as a consequence of the entropy gradient

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The evolution of all material systems whose structure is controlled by electromagnetic and gravitational (inertial) forces is irreversible.

Time evolution is formulated using concepts defined by the laws of balance, both in terms of phenomenology and quantum mechanics:

- energy and mass
- momentum
- angular momentum
- entropy

Specific concepts of quantum mechanics are taken into account.

specific concepts of quantum mechanics

- leptons and quarks are components of matter,
- bosons mediate force interaction

Systems dominated by gravitational interaction (black holes, stars, planets, etc.) as well as those dominated by electromagnetic interaction exist in a state of thermodynamic non-equilibrium. This thermodynamic non-equilibrium is maintained by a negative entropy flux.

Abstract: Connection between classical mass point mechanics and continuum thermodynamics

The direct correlation between classical mechanics of material points (Lagrange principle of classical mechanics) and classical continuum mechanics can be established when the existence of a trajectory and a friction force are added.

- Except for those parts of the universe where the density of matter is exceptionally high (in regions around the Schwarzschild radius, i.e. black holes, neutron stars, etc.), the universe can be considered Euclidean or pseudo-Euclidean and **the methods of classical solid and fluid mechanics can be used.**
- The Lagrangian of classical mechanics of continuous systems is extended by an additional fictitious frictional energy arising during the movement of a material point. *It could also be that mysterious dark energy.*
- The extremum of the functional created in this way occurs under the condition of conservation of total enthalpy and balance of momentum where **the gravity and acceleration are represented by entropy gradient.**
- By eliminating the corresponding fictitious force, the inertia reveals and **connection between the acceleration and temperature is a consequence.**
- The consequences of the extreme of **the total enthalpy are conditions of thermodynamic stability** of the movement of material points.

General theory of relativity (or Einstein's theory of gravity), is the geometric theory of gravitation

Basic principles:

- Equivalence principle: gravitational and inertial forces have the same nature.
- General principle of relativity: Physical laws have the same form in all reference frames (principle of invariance).
- Space-time is curved: Mass and energy curve space and time, which is reflected in the path of objects and light. The metric tensor of the corresponding space-time is described by Einstein's equation

$$R^{\alpha\beta} - \frac{1}{2}Rg^{\alpha\beta} + \underbrace{(\Lambda g^{\alpha\beta})}_{\text{cosmological term}=0} = \frac{8\pi G}{c^4} T^{\alpha\beta} \quad \text{for } G = 6.674 \cdot 10^{-11} [\text{m}^3\text{kg}^{-1}\text{s}^{-2}]$$

- $g^{\alpha\beta}$ for $\alpha, \beta = 0, 1, 2, 3 \dots$ metric tensor...for 3D it has the same interpretation as the Cauchy deformation tensor
- $R^{\alpha\beta} \dots$ Ricci curvature tensor-measure expressing the local change of the geometry of a given metric tensor from the geometry of ordinary Euclidean space
- $R = g^{\gamma\delta} R_{\gamma\delta} \dots$ scalar curvature
- $\Lambda g^{\alpha\beta} \dots$ cosmological term may represent dark energy
- $T^{\alpha\beta} \dots$ stress-energy tensor, for $T^{\alpha\beta}_{;\beta} = 0$, for $\alpha = 0$ balance of mass and for $\alpha = 1, 2, 3$ balance of momentum.

Solution of Einstein's equations in the vicinity of a spherically symmetric body of mass M (Karl Schwarzschild, 1916).

Solving the Einstein equation

$$R^{\alpha\beta} - \frac{1}{2}Rg^{\alpha\beta} = 0 \quad \text{in coordinates } \xi^\alpha = (t, r, \theta, \phi)$$

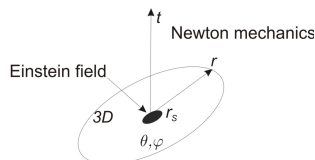
$$ds^2 = g_{\alpha\alpha} d\xi^{\alpha 2} = c^2 \left(1 - \frac{r_s}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{r_s}{r}} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

$$\text{where } r_s = \frac{2GM}{c^2} \text{ is Schwarzschild radius}$$

Schwarzschild radius for Earth is 9mm and for Sun is $r_s = 3\text{km}$.

For almost all astrophysical objects, $r_s/r \rightarrow 0$ holds. Except for regions of $r_s/r \leq 1$ (black hole region) has a cosmic space a pseudo-Euclidean metric, e.g.: Minkowski space,

$$ds^2 = \eta_{\alpha\alpha} d\xi^{\alpha 2} = c^2 dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

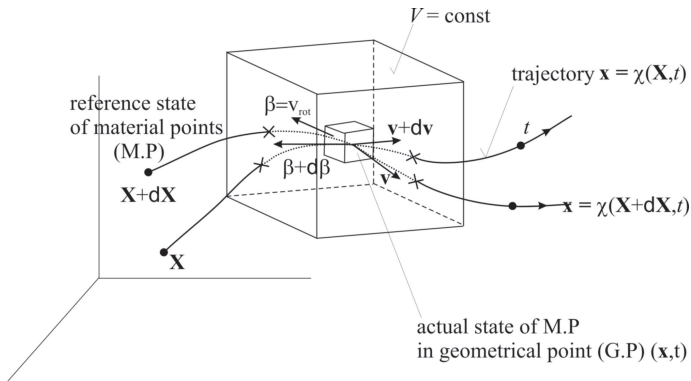


Under this assumption, we can move on to the so-called Newton limit $g_{00} = -\left(1 - \frac{2\Phi_G}{c^2}\right)$ and Einstein's equation transforms into Laplace's equation

$$\Delta \Phi_G = 4\pi G \rho_M \quad \text{or} \quad \Phi_G = -\frac{GM}{r} \quad \text{for} \quad \mathbf{g} = -\nabla \Phi_G = -\frac{GM}{r^2} \mathbf{e}_r$$

The above brief consideration of general relativity was intended to show that outside the region of strongly distorted space-time (the vicinity of massive matter densities, black holes, etc.) we can use classical mechanics.

Variation of the Material Point trajectory



At the G.P. (x, t) has the M.P. X the velocity $v(x, t)$ (There is a velocity field around the G.P.) and its interaction force is $\dot{\beta}(x, t)$. The trajectory becomes irrelevant and the field, e.g. velocity, temperature, entropy, etc., is essential to describe the change (evolution).

- Seliger R.L., Whitham G.B.: Variational Principles of Continuum mechanics, *Proc. Roy Soc.* **A.305**, 1-25, 1968.
- Maršík F., Trávníček Z., Antošová Z.: The connection between the principle of the least action and the thermodynamic condition of stability. 29th International Conference ENGINEERING MECHANICS 2023, Milovy, Czech Republic, May 9-11, 2023, <https://doi.org/10.21495/em2023-159>

The principle of the least action or the **Hamilton principle**.

$$S(\mathbf{v}, \beta, \mathbf{x}) = \int_{t_0}^{t_1} \int_V \rho \left[\frac{\mathbf{v}^2}{2} - \Phi(\mathbf{x}) - u(\rho(\mathbf{x}, t), s(\mathbf{x}, t)) - \mathbf{X}(\mathbf{x}, t) \dot{\beta}(\mathbf{x}, t) \right] dv dt$$

- $\frac{\mathbf{v}^2}{2}$ is the kinetic energy of material point
- Φ is the potential energy,
- $\mathbf{X} \dot{\beta} = \sum_{L=1}^3 X^L(\mathbf{x}, t) \dot{\beta}_L(\mathbf{x}, t) = x^i \dot{\beta}_i(\mathbf{x}, t)$ is the energy of interaction with surroundings at the G.P. $(\mathbf{x}, t)$
- $u(\rho((\mathbf{x}, t)), s((\mathbf{x}, t)))$ is the internal energy of the material point (M.P.)
- For solid body is $u(\mathbf{e}((\mathbf{x}, t)), s((\mathbf{x}, t)))$ where $\mathbf{e}$ is the Euler deformation tensor.

The independent quantities are

- the trajectory $\mathbf{x} = \chi(\mathbf{X}, t)$. of the M.P., $\mathbf{X}$ is the initial position of the M.P.
- the friction velocity $\beta(\chi, t)$.

We suppose that each material point has the initial position $\mathbf{X}$ -at some instant t_0 . Its motion is described by the trajectory $\mathbf{x} = \chi(\mathbf{X}, t)$ (with the inverse mapping $\mathbf{X}\mathbf{x} = \mathbf{x}\mathbf{X} = \mathbf{I}$) and the motion of all material points materializes in the velocity field $\mathbf{v}(\mathbf{x}, t)$.

The variations (fluctuations) of the M.P. trajectory

The variations (fluctuations) of the M.P. trajectory $\delta\tilde{x}^i(\mathbf{X}, t) = \tilde{x}^i(\mathbf{X}, t) - \tilde{x}_0^i(\mathbf{X}, t)$ and friction velocity (material parameter), $\delta\beta_L(\mathbf{X}, t) = \beta_L(\mathbf{X}, t) - \beta_{L,0}(\mathbf{X}, t)$

$$\begin{aligned}
 \delta S &= \delta \int_{t_0}^{t_1} \int_V \rho \left[\overbrace{\frac{\mathbf{v}^2}{2} - \Phi(\mathbf{x}) - u(\rho(\mathbf{x}, t), s(\mathbf{x}, t)) - \mathbf{X}(\mathbf{x}, t) \dot{\beta}(\mathbf{x}, t)}^{l(\mathbf{x}, \mathbf{v}, \beta)} \right] dv dt \\
 &= \int_{t_0}^t \int_V (\rho \delta l + l \delta \rho) dv dt = \int_{t_0}^t \int_V \left\{ \rho \left[\overbrace{\left[-\dot{\beta}_L \frac{\partial X^L}{\partial x^i} - \frac{\partial \Phi}{\partial x^i} - T \frac{\partial s}{\partial x^i} \right]}^{\text{Balance of momentum}} \delta \tilde{x}^i + \overbrace{\left(l - \frac{\rho}{\rho} \right) \frac{\partial \rho}{\partial x^i} \delta \tilde{x}^i}^{\text{Balance of energy}} \right. \right. \\
 &\quad \left. \left. + \overbrace{\left[\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v^i)}{\partial x^i} \right] X^L \delta \beta_L}^{\text{Balance of mass}} + \overbrace{\rho \left(v_i - X^L \frac{\partial \beta_L}{\partial x^i} \right) \frac{\partial \delta \tilde{x}^i}{\partial t}}^{\text{Velocity field def.}} \right. \right. \\
 &\quad \left. \left. + \rho \overbrace{\left(\frac{\partial X^L}{\partial t} + v^j \frac{\partial X^L}{\partial x^j} \right) \delta \beta_L}^{\text{Fixed position of M.P.}} \right\} dv dt - \int_V \rho X^L \delta \beta_L dv \Big|_{t_0}^t - \int_{t_0}^t \int_{\partial V} \rho v^i X^L \delta \beta_L da_i dt = 0
 \end{aligned}$$

The general laws of mechanics and thermodynamics have to be valid for all material points, i.e. for all trajectories which intersect this point $\mathbf{x} = \tilde{\mathbf{x}}(\mathbf{X}, t)$.

Necessary extremum conditions I

Unknown $\tilde{x}_0^i(\mathbf{X}, t)$ is the optimum (reference) trajectory, which satisfy extremum condition of the **Hamilton principle**. Velocity variation $\delta v^i = \delta \dot{\tilde{x}}^i$ depends on the $\delta \tilde{x}^i(\mathbf{X}, t)$.

Necessary condition for velocity field is

$$v_i = X^L \frac{\partial \beta_L}{\partial x^i} \quad \text{or} \quad v^i v_i = v^2 = X^L v^i \frac{\partial \beta_L}{\partial x^i} = X^L \dot{\beta}_L - X^L \frac{\partial \beta_L}{\partial t}$$

Velocity field can be divided into

$$v_i = X^L \frac{\partial \beta_L}{\partial x^i} = \overbrace{\frac{\partial (X^L \beta_L)}{\partial x^i}}^{\varphi(\mathbf{x}, t)} - \beta_L \frac{\partial X^L}{\partial x^i} = v_{i \text{ pot}} + v_{i \text{ rot}}, \quad v_{i \text{ pot}} = \frac{\partial \varphi}{\partial x^i}, \quad v_{i \text{ rot}} = \beta_i = -\beta_L \frac{\partial X^L}{\partial x^i}$$

The vorticity **rot** $\mathbf{v}$ is generated by friction.

The lagrangian can be written as

$$I = \frac{v^2}{2} - \phi - u - X^L \dot{\beta}_L = \frac{v^2}{2} - \phi - u - v^2 - X^L \frac{\partial \beta_L}{\partial t} = -X^L \frac{\partial \beta_L}{\partial t} - h_t + \frac{p}{\rho}$$

The laws of **conservation of the energy in the G.P.** is

$$I - \frac{p}{\rho} = -X^L \frac{\partial \beta_L}{\partial t} - h_t = 0 \quad \dots \text{energy conservation,} \quad \text{for} \quad h_t = \frac{v^2}{2} + u + \frac{p}{\rho} + \Phi$$

Total enthalpy h_t depends on the local friction force, only i.e., $-X^L \frac{\partial \beta_L}{\partial t} = h_t$. For the local stationary friction field, i.e., $\partial(\beta)/\partial t = 0$, the total enthalpy of the fluid is constant. Analogous to the classical mechanics of a mass point, when in the ideal case the sum of the kinetic and potential energy is constant.

The balance of momentum can be reformulated in the measurable quantities as follows

$$-\dot{\beta}_L \frac{\partial \tilde{X}^L}{\partial x^i} - \frac{\partial \Phi}{\partial x^i} - T \frac{\partial s}{\partial x^i} = 0 \quad \text{or} \quad \frac{\partial v_i}{\partial t} - (\mathbf{v} \times \mathbf{rot} \mathbf{v})_i = -\frac{\partial h_t}{\partial x^i} + T \frac{\partial s}{\partial x^i} + \frac{\partial \Phi}{\partial x^i}$$

where the unknown friction force $\dot{\beta}$ was eliminated. The correspondence to **the Crocco theorem** is evident

$$-(\mathbf{v} \times \mathbf{rot} \mathbf{v})_i = -\frac{\partial h_t}{\partial x^i} + T \frac{\partial s}{\partial x^i} + \frac{\partial t_{dis}^{li}}{\rho \partial x^i} \quad \text{Crocco theorem}$$

grad h_t , grads and volume forces generate the vorticity **rot $\mathbf{v}$** . This formulation of the law of conservation of momentum in classical continuum mechanics is only possible when the concept of entropy is introduced in the formulation of the law of energy balance - **I. Laws of thermodynamics.**

The origin of gravity and inertia.

The momentum balance resulting from the variational principle is

$$\begin{aligned} \frac{\partial v_i}{\partial t} - (\mathbf{v} \times \mathbf{rot} \mathbf{v})_i &= -\frac{\partial h_t}{\partial x^i} + T \frac{\partial s}{\partial x^i} + \frac{\partial \Phi}{\partial x^i} \\ &= -\frac{\partial}{\partial x^i} \left(\frac{\mathbf{v}^2}{2} \right) - \frac{\partial h}{\partial x^i} + T \frac{\partial s}{\partial x^i} + \underbrace{-\frac{\partial \Phi}{\partial x^i} + \frac{\partial \Phi}{\partial x^i}}_{=0} \\ \underbrace{\frac{\partial v_i}{\partial t} + \frac{\partial}{\partial x^i} \left(\frac{\mathbf{v}^2}{2} \right) - (\mathbf{v} \times \mathbf{rot} \mathbf{v})_i}_{\dot{v}_i \dots \text{inertia of 1 kg mass}} &= -\frac{\partial h}{\partial x^i} + \underbrace{T \frac{\partial s}{\partial x^i}}_{\text{gravity}} = -\frac{\partial h}{\partial x^i} - \underbrace{\frac{\partial \Phi_s}{\partial x^i}}_{g_i} \end{aligned}$$

or $\dot{v}_i = - \underbrace{\frac{\partial h}{\partial x^i}}_{=0 \dots \text{isoentropic flow}} + T \frac{\partial s}{\partial x^i} = -\frac{\partial \Phi(s)}{\partial x^i}$ -equivalence of inertial and gravitational mass

mass $\equiv$ energy then $h(s, p) [\text{Jkg}^{-1} = \text{m}^2\text{s}^{-2}] = \overbrace{u(\rho, s)}^{\text{internal energy}} + \overbrace{\frac{p}{\rho}}^{\text{pressure}} = \text{const} \sim c^2$

Potential energy $\Phi(s)$ depends on entropy and the Newton's law of gravitation of mass M in spherical coordinates has the form

$$\mathbf{g} = -\text{grad} \Phi(s) = -\frac{\partial}{\partial r} \left(\overbrace{\frac{-GM}{r}}^{\Phi(s)} \right) \mathbf{e}_r = -\frac{GM}{r^2} \mathbf{e}_r = -g \mathbf{e}_r, \quad \text{for } G = 6.674 \cdot 10^{-11} [\text{m}^3\text{kg}^{-1}\text{s}^{-2}]$$

Connecting general relativity with quantum mechanics using entropy.

The quantum mechanical description of matter should be used for objects whose geometric dimensions are comparable to the Compton length or smaller. Assuming that the geometric dimension of the material point (M.P.) is comparable to the Compton length

$$r_s = \frac{2GM}{c^2} = \frac{\lambda_C}{2\pi} = \frac{h}{2\pi mc} \quad \text{then for } M = m = m_P = \sqrt{\frac{hc}{2\pi G}} = 2.18 \cdot 10^{-8} \text{ kg}$$

for comparison mass of proton is $m_P = 1.673 \cdot 10^{-27} \text{ kg}$ and moreover $r_s \frac{\lambda_C}{2\pi} = l_P^2$ where

$$l_P = \sqrt{\frac{hG}{2\pi c^3}} = 1.616 \cdot 10^{-35} \text{ m is the Planck length.}$$

Assuming the equivalence of mass and energy, there is c^2 energy (joules) in one kg of mass. Then the **temperature and entropy gradient are the causes of both inertia and gravity**. In equilibrium, the volume force g due to gravity must be equal to the volume force due to inertia a , i.e.:

$$\begin{aligned} \dot{v}_i = a_i &= -\frac{\partial \Phi(s)}{\partial x^i} = -g_i = T \frac{\partial s}{\partial x^i} \quad \text{or for the case of spherical symmetry} \\ a \mathbf{e}_r &= -\frac{\partial \Phi(s)}{\partial r} \mathbf{e}_r = -g \mathbf{e}_r = T \frac{\partial s}{\partial r} \mathbf{e}_r \end{aligned}$$

The question is how to determine temperature T and entropy gradient $\frac{\partial s}{\partial r}$?

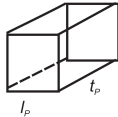
We start from Newton's law of gravity. The gravitational acceleration of a Planck mass m_P at a distance of the Planck length l_P is

$$-g_P \mathbf{e}_r = -\frac{Gm_P}{l_P^2} \mathbf{e}_r = -T_P \left. \frac{\partial s}{\partial r} \right|_P \mathbf{e}_r = -\sqrt{\frac{2\pi c^7}{hG}} \mathbf{e}_r$$

Relationship between acceleration and entropy gradient in Planck units

For $h(s, p) = \text{const}$, the inertia $\dot{\mathbf{v}}$ is a consequence of an existence of the friction energy $x^i \dot{\beta}_i(\mathbf{x}, t)$ and the gravity $\mathbf{g} = \mathbf{grad}\Phi$ is a consequence of the entropy gradient $\mathbf{grad}s$. Assuming, the friction force is steady (i.e., $\partial\beta_i/\partial t = 0$), the inertia is equivalent to the gravitational force.

Space-time element-- 4d



$$l_p = \sqrt{\frac{hG}{2\pi c^3}} = 1.616 \cdot 10^{-35} \text{ m} \quad \text{Planck length}$$

$$t_p = \sqrt{\frac{hG}{2\pi c^5}} = 5.4 \cdot 10^{-44} \text{ s} \quad \text{Planck time}$$

$$m_p = \sqrt{\frac{hc}{2\pi G}} = 2.18 \cdot 10^{-8} \text{ kg} \quad \text{Planck mass}$$

$$T_p = \sqrt{\frac{hc^5}{2\pi G k_B^2}} = 1.416 \cdot 10^{32} \text{ K} \quad \text{Planck temperature}$$

For the Planck dimensions of space-time

$$\begin{aligned} \dot{\mathbf{v}}_P &= -g_P \mathbf{e}_r = -\frac{Gm_P}{l_p^2} \mathbf{e}_r = -\frac{G\sqrt{\frac{hc}{2\pi G}}}{\frac{hG}{2\pi c^3}} \mathbf{e}_r = -\sqrt{\frac{2\pi c^7}{hG}} \mathbf{e}_r \text{ or for acceleration } \frac{c}{t_p} \mathbf{e}_r = a_P \mathbf{e}_r = \sqrt{\frac{2\pi c^7}{hG}} \mathbf{e}_r \\ &= T_P \left. \frac{\partial s}{\partial r} \right|_P \mathbf{e}_r \cdots \text{acceleration depends on entropy gradient} \end{aligned}$$

$$\dot{\mathbf{v}} = a \mathbf{e}_r = T \frac{\partial s}{\partial r} \mathbf{e}_r = -g \mathbf{e}_r \quad \text{if space is in equilibrium} \quad \left. \frac{\partial s}{\partial r} \right|_P = \frac{\partial s}{\partial r} = \text{const} \leq 0 \text{ for } r \in \langle l_p, r \rangle$$

entropy is additive function of space and $\frac{\partial s}{\partial r}$ is positive for acceleration and negative for gravity.

$$\text{then } \frac{\partial s}{\partial r} = \frac{a}{T} = -\frac{g}{T} = 2\pi \frac{a_P}{T_P} = 4\pi^2 \frac{k_B C}{h} = 2.464 \cdot 10^{20} \text{ mK}^{-1} \text{s}^{-2} = \text{const}, \Rightarrow \text{Unruh-Hawking relation}$$

$2\pi \cdots$ added due to quantum nature. For $g = 9.81 \text{ m s}^{-2}$ we obtain $T = 4 \cdot 10^{-20} \text{ K}$

The gravity and temperature of a body of mass M .

To determine the gravitational acceleration induced by mass M on the surface of a sphere of radius R , we can use the Newton gravitation law and the formula $\frac{\partial s}{\partial r} = -4\pi^2 \frac{k_B c}{h}$

$$\mathbf{g}(R) = -\frac{\partial}{\partial r} \left(\overbrace{\left(\frac{-GM}{r} \right)}^{\Phi(s)} \right) \Big|_R \mathbf{e}_r = -\frac{\partial \Phi(s)}{\partial s} \Big|_R \frac{\partial s}{\partial r} \mathbf{e}_r = T \frac{\partial s}{\partial r} \mathbf{e}_r = -4\pi^2 T \frac{k_B c}{h} \mathbf{e}_r$$

$$\text{or } \frac{GM}{R^2} = 4\pi^2 T \frac{ck_B}{h} \quad \text{and the consequence is the temperature} \quad T = -\frac{\partial \Phi(s)}{\partial s} \Big|_R = \frac{hGM}{4\pi^2 ck_B R^2}$$

An alternative method for determining temperature is based on Verlinde's holographic principle (E. Verlinde (2010)), which assumes that the temperature T depends on the mass M inside the sphere of a radius R and on the number of elements l_p^2 on the surface $4\pi R^2$. By the equipartition theorem for one degree of freedom of particles (elements of surface) l_p^2 . We assume that these elements can only vibrate in the direction of the normal.

$$M = \frac{E}{c^2} = \frac{Nk_B T}{2c^2} \quad \text{for } N = \frac{4\pi R^2}{l_p^2}$$

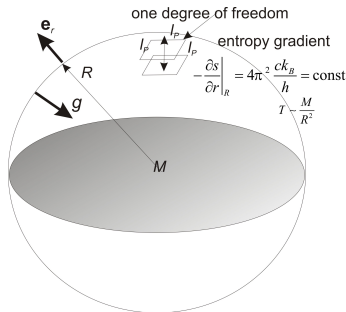
and corresponding temperature is

$$T = \frac{2c^2 l_p^2 M}{4\pi k_B R^2} = \frac{h}{4\pi^2 ck_B} \frac{GM}{R^2}$$

Then, the gravitational acceleration is

$$\mathbf{g}(R) = T \frac{\partial s}{\partial r} \Big|_R \mathbf{e}_r = -\frac{GM}{R^2} \mathbf{e}_r$$

and the Newton gravity law is the consequence



Distribution of entropy around the Schwarzschild radius - around the event horizon.

The entropy generated by gravity is

$$\frac{\partial s}{\partial r} = -4\pi^2 \frac{k_B c}{h} \quad \text{for } r \in (r_S, \infty), \varphi, \theta = \text{const}, r_S = \frac{2GM}{c^2}$$

and for the body mass M we have

$$dS = -4\pi^2 \frac{k_B c M}{h} dr, \text{ or } S(r) = S_0 - 4\pi^2 \frac{k_B c M}{h} (r - r_S) = S_{BH} \left(2 - \frac{r}{r_S}\right),$$

$$S_0 = S_{BH} = \frac{8\pi^2 k_B G M^2}{ch} \dots \text{Bekenstein-Hawking formula}$$

The S_{BH} relation is also the entropy of a non-rotating black hole. From this relation it is clear that the entropy outside the black hole, specifically for $r > 2r_S$, is negative in the stationary state. If there is any matter in the vicinity of the black hole, it will be attracted so that equilibrium is again established with some other external negative entropy. **The black hole therefore ensures its existence (stability) by absorbing negative entropy.** Analogous principle of the existence of non-equilibrium (biological) systems in the field of electromagnetic interactions. To satisfy the condition that the entropy at the center of a black hole is zero, we have to use the relation for

$$\frac{\partial s}{\partial r} = 4\pi^2 \frac{k_B c}{h} \quad \text{for } r \in (0, r_S), \varphi, \theta = \text{const}, r_S = \frac{2GM}{c^2}$$
$$S(r) = S_0 + 4\pi^2 \frac{k_B c M}{h} (r - r_S) = S_{BH} \frac{r}{r_S} \quad \text{for } S(r=0) = 0.$$

Comparison with E. Verlinde's concept of entropic gravity.

- **In this concept**, the basic starting point is to determine the optimal trajectory of a mass point in the classical sense. This mass point has a non-zero mass and temperature, and in addition, the space is filled with additional frictional energy (it may be dark energy). The consequence is the onset of inertia and a constant entropy gradient $\frac{\partial s}{\partial r} = 4\pi^2 \frac{k_B c}{h}$. The magnitude of the acceleration a and gravitational acceleration g depends on the temperature $a = g = 4\pi^2 T \frac{k_B c}{h}$. **The temperature depends on the size of the mass and the distance from the center $T \sim M/R^2$ and is constant at a given point in spacetime.**
- **In 2009, E. Verlinde showed** that the laws of gravity may be derived by assuming a form of the holographic principle and the laws of thermodynamics. This may imply that gravity is not a true fundamental force of nature (like e.g. electromagnetism), but instead is a consequence of the universe striving to maximize entropy. He also admits the force effects of dark energy.

Born	Erik Peter Verlinde 21 January 1962 (age 63) Woudenberg, Netherlands
Nationality	Dutch
Alma mater	Utrecht University
Known for	Verlinde algebra Matrix string theory Entropic gravity Emergent gravity
Awards	Spinoza Prize (2011)
	Scientific career
Fields	Theoretical physics
Institutions	Princeton University University of Amsterdam



We determine the magnitude of the gravitational acceleration on the magnitude of the mass M , e.g., for Earth. We will use the equipartition theorem for one degree of freedom of particles (elements of surface l_P^2) on some surface $4\pi R^2$ enclosing mass M , it is

$$M = \frac{E}{c^2} = \frac{Nk_B T}{2c^2} \quad \text{for} \quad N = \frac{4\pi R^2}{l_P^2}$$

Under these assumptions and using the Unruh-Hawking relation $T = gT_P/(2\pi a_P)$ we have

$$M = \frac{E}{c^2} = \frac{Nk_B T_P g}{4\pi c^2 a_P} = \frac{k_B T_P g R^2}{c^2 a_P l_P^2} = \frac{g R^2}{G}$$

The gravitational acceleration of the Earth at a distance R is

$$g = \frac{GM}{R^2}$$

This relationship is valid for any matter.

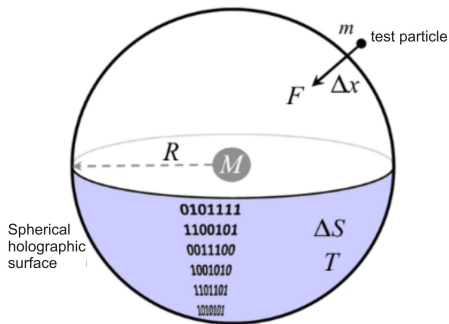
Verlinde gravitation and inertia

An interesting connection with the holographic scenario of the universe (Erik Verlinde, 2010). Viz. Petr Kulháněk, https://www.aldebaran.cz/bulletin/2011_4_ver.php

The basic relationship for the entropic force is the energy balance

$$\Delta Q \leq T \Delta S = \underbrace{\Delta U}_{=0} + p \underbrace{\Delta V}_{A \Delta x} = F \Delta x$$

Let us now assume a projection surface in the shape of the surface of a sphere. This is not to the detriment of the generality, any enclosed projection area can be composed of elementary rectangles or polygons, and the area will always increase with the square of the size of the area.



Let's assume that all information is localized on this surface, i.e. the holographic principle applies. Information on the surface of the sphere, the properties of matter M inside the sphere as well as the properties of space and time are determined. A small test particle of mass m moves near the sphere.

As a result of the displacement of the particle with mass m , the entropy of the holographic surface changes. We choose the smallest possible entropy change $\Delta S = 2\pi k_B$ (More realistic would probably be $\Delta S = k_B \ln 2 = 0.693 k_B$) and the smallest possible displacement of a particle with momentum mc is according to uncertainty relation $\Delta x = h/(2\pi mc)$. Then

$$\Delta S = 2\pi k_B = \frac{(2\pi)^2 k_B mc}{h} \Delta x \sim m \Delta x$$

We determine the temperature of a particle that has acceleration a from the Unruh-Hawking relation

$$T = \frac{a}{2\pi a_P} T_P$$

where a_P , T_P are respectively the Planck acceleration and the Planck temperature. **Temperature does not exist in an inertial frame.**

$$T \Delta S = \frac{a}{2\pi a_P} T_P \frac{(2\pi)^2 k_B mc}{h} \Delta x = ma \Delta x = F \Delta x$$

Temperature causes acceleration and its consequence is the law of inertia of matter.

$$F = T \text{grad} S = ma$$

Assuming that the holographic principle holds on the boundary is a storage device for information. By definition, each basic bit occupies one unit cell of volume (surface) $l_P^2 = hG/(2\pi c^3)$. So, it's on the surface

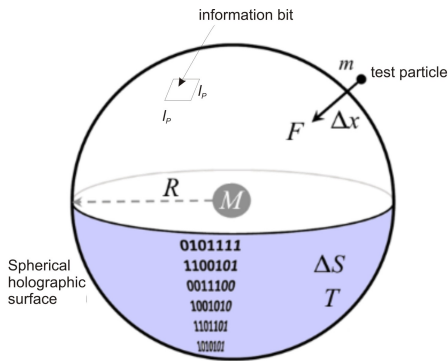
$$N = \frac{4\pi R^2}{l_P^2} \quad \text{bits}$$

The total energy contained in the volume of the sphere is given by the mass M and is equals $E = Mc^2$. This energy manifests itself on the surface according to the equipartition law by the temperature T

$$E = Mc^2 = \frac{1}{2} N k_B T \quad \text{or} \quad T = \frac{2Mc^2}{k_B N}$$

After substituting into the energy balance equation

$$F \Delta x = T \Delta S = \frac{2Mc^2}{k_B \frac{4\pi R^2}{l_P^2}} \frac{(2\pi)^2 k_B m c}{h} \Delta x = G \frac{mM}{4\pi R^2} \Delta x \quad \text{or} \quad F = G \frac{mM}{4\pi R^2} \dots \text{gravity}$$



Balance of energy-I. Law of thermodynamics and Balance of entropy-II. Law of thermodynamics

The **I. Law of Thermodynamic** formulated with the total enthalpy is given by

$$\rho \dot{h}_t - \frac{\partial p}{\partial t} = -\frac{q^k}{T} \frac{\partial T}{\partial x^k} + \frac{\partial(t_{dis}^{ki} v_i)}{\partial x^k} \quad \text{for} \quad h_t = \frac{v^2}{2} + u + \frac{p}{\rho} + \Phi$$

Definition of entropy S follows from the global form of the **II. Law of thermodynamics** formulated as the balance of the total entropy

$$\dot{S} - J_D(S) = P(S) \geq 0, \quad \text{for} \quad J_D(S) = - \int_{\partial V} \frac{q^i}{T} da_i, \quad P(S) = \int_V \sigma(s) d\nu \geq 0$$

where $J_D(S)$ is entropy flux, $P(S) \geq 0$ is total entropy production and $\sigma(S) \geq 0$ is the density of entropy production. Eliminating heat flux $\mathbf{q}$ by I. Law of thermodynamic we get II. Law of thermodynamic in the local form, so called **dissipative inequality**

$$\pi = T\sigma(S) = \rho \left(\underbrace{T\dot{s} + \frac{1}{\rho} \frac{\partial p}{\partial t} - \dot{h}_t}_{=0 \dots \text{entropy definition}} - \frac{q^k}{T} \frac{\partial T}{\partial x^k} + \frac{\partial(t_{dis}^{ki} v_i)}{\partial x^k} \right) \geq 0,$$

The thermodynamic stability criterion can be formulated by h_t . The entropy is the function of h_t, p and the thermodynamic inequality (II. Law of thermodynamics) is for $\pi < 0$ violated and the instability can occurs.

Definition of entropy for processes with convection

$$T\dot{s} = \dot{h}_t - \frac{1}{\rho} \frac{\partial p}{\partial t} \quad \text{for} \quad h_t = \frac{v^2}{2} + u + \frac{p}{\rho} + \Phi$$

switches to the usual Gibbs definition of entropy for the case of zero velocity of the material point

$$T\dot{s} = v^j \dot{v}_j + \dot{u} + \frac{1}{\rho} \dot{p} + p \left(\frac{\dot{1}}{\rho} \right) + \dot{\Phi} - \frac{1}{\rho} \frac{\partial p}{\partial t} \quad \text{for} \quad \dot{() } = \frac{\partial ()}{\partial t} + v^i \frac{\partial ()}{\partial x^i}, \text{ i. e. convective derivative}$$

for $v^i = 0$ for $i = 1, 2, 3$ we obtain classical Gibbs definition of entropy

$$T \frac{\partial s}{\partial t} = \frac{\partial u}{\partial t} + p \frac{\partial}{\partial t} \left(\frac{1}{\rho} \right)$$

We remind you that the Earth's gravitational field is stationary $\partial\Phi/\partial t = 0$, as is the friction field $\partial(\beta)/\partial t = 0$, see Necessary condition II.

The specific total enthalpy is a function of the entropy and the pressure, i.e. $h_t = h_t(s, p)$ and satisfies the general relation

$$\dot{h}_t = T\dot{s} + \frac{1}{\rho} \frac{\partial p}{\partial t}.$$

All dissipative processes in the system, which is in the thermodynamic equilibrium, which we will call a **reference state** (denoted by subscript "0"), are zero. Considering the rapid temporal change of fluctuations, this reference state changes only very slowly in time. For such a reference state, $h_{t0} = h_t(s_0, p_0)$ has a dissipation extremum, i.e. is equal to zero

$$T_0\dot{s}_0 - \dot{h}_{t0} + \frac{1}{\rho_0} \frac{\partial p_0}{\partial t} = 0, \quad \tilde{\pi} = \tilde{\pi}_0 = 0$$

Now, let us suppose that the system deviates ($s = s_0 + \delta s$, $p = p_0 + \delta p$) from this reference state and its total enthalpy changes. The deviations (fluctuations) δs , δp generate a dissipation $\tilde{\pi} \geq 0$, i.e.

$$T_0\dot{s} - \dot{h}_t + \frac{1}{\rho_0} \frac{\partial p}{\partial t} = \tilde{\pi}_0 + \tilde{\pi} = \tilde{\pi} \geq 0$$

Expanding the total enthalpy around the reference state and inserting this into the dissipation inequality gives

$$\begin{aligned}
 h_t(s, p) &= h_{t0} + dh_{t0} + \frac{1}{2}d^2h_{t0} + \dots, \\
 -\frac{\dot{d}h_{t0}}{dt} + T_0\frac{\dot{\delta s}}{\delta s} + \frac{1}{\rho_0}\frac{\partial \delta p}{\partial t} &= \tilde{\pi}_0 = 0 \quad \text{extremum condition for } \tilde{\pi} \\
 -\frac{1}{2}\frac{\dot{d^2h_{t0}}}{dt^2} &= \tilde{\pi} = \frac{q^k}{T}\frac{\partial T}{\partial x^k} + \frac{\partial(t_{dis}^{ki}v_i)}{\partial x^k} \geq 0 \quad \text{stability condition}
 \end{aligned}$$

This inequality is an alternative form of the **thermodynamic condition of the stability for processes with convection**. The dissipation $\tilde{\pi}$ is the second order term in the total enthalpy change. The derivation is similar to the derivation of the classical Gibbs stability conditions. Thermodynamic fluid systems with dominant convection are stable when the inequalities both for the second differential $d^2h_t \geq 0$ and for its material derivative $\frac{\dot{d^2h_t}}{dt^2} \leq 0$, are satisfied.

$d^2h_t \geq 0$ is the Lyapounov function of the stability for processes with convection.

- Maršík F., Trávníček Z., Weigand B., Seibold F., Antořová Z.: Swirl flow stability: thermodynamic analysis and experiments. *Continuum Mech. Thermodyn.*, <https://doi.org/10.1007/s00161-024-01303-6>

Thermodynamic approach to fluid stability

Thermo-visco-elastic system

$$q^i = -\lambda(\rho, T)\delta^{ik}\frac{\partial T}{\partial x^k}, \quad t_{dis}^{ki} = \mu(\rho, T)\left(\frac{\partial v^k}{\partial x^i} + \frac{\partial v^i}{\partial x^k} - \frac{2}{3}\delta^{ki}\text{div}\mathbf{v}\right)$$

For the simplified flow, $v_\varphi = v_\varphi(r)$, $T = T(r)$ has **thermodynamic condition of the stability** the form

$$\tilde{\pi} = \frac{\lambda}{T}\left(\frac{\partial T}{\partial r}\right)^2 + \mu\left[\left(\frac{\partial v_\varphi}{\partial r}\right)^2 + v_\varphi\frac{\partial^2 v_\varphi}{\partial r^2} - \frac{v_\varphi}{r}\frac{\partial v_\varphi}{\partial r}\right] + \frac{\partial \mu}{\partial r}\left(v_\varphi\frac{\partial v_\varphi}{\partial r} - \frac{v_\varphi^2}{r}\right) \geq 0 \quad (1)$$

Considering that the thermal conductivity $\lambda > 0$ and viscosity $\mu > 0$ coefficients are always positive coefficient the associated **blue terms** are positive too and will only have a stabilizing effect. We therefore focus only on the influence of the flow field $v_\varphi(r)$, where the viscosity μ plays dominant role.

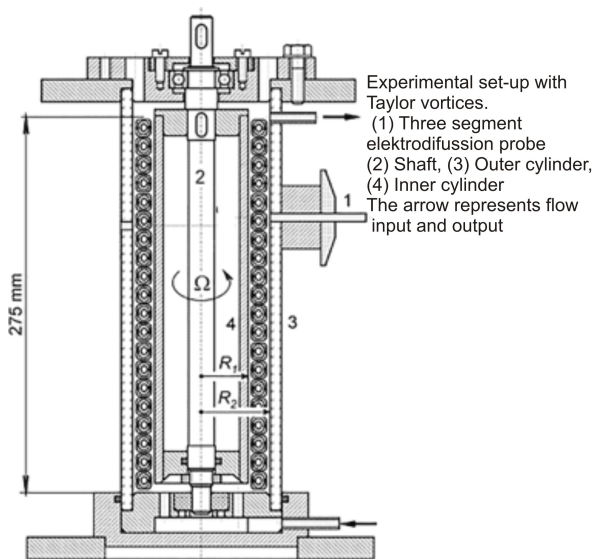
$$\begin{aligned} v_\varphi &= \omega r \quad \text{then} \quad \tilde{\pi} = 0 \quad \text{solid body vortex with angular velocity } \omega \\ v_\varphi &= \Gamma/r \quad \text{then} \quad \tilde{\pi} = 4\Gamma^2/r^4 \geq 0 \quad \text{potential vortex} \end{aligned} \quad (2)$$

The solid body vortex is on the onset of stability and the potential vortex with circulation Γ is stable. Therefore, the set of solutions of the Navier-Stokes equations containing a potential vortex exhibits a singularity at $t \rightarrow \infty$ and $r \rightarrow 0$.

To show the generality of the thermodynamic stability condition the following problems are studied:

- classical Taylor-Couette flow between two cylinders
- annular swirling jet transformation
- tropical cyclones-Rankine vortex approximation

Taylor-Couette flow



DOI:10.1371/journal.pone.0212728Corpus ID: 92999853

Experimental investigation of Taylor-Couette-Poiseuille flow at low Taylor and Reynolds numbers

M. Kristiawan, M. El Hassan, Alioune Faye, V. Sobolik less

Published 3 April 2019

Engineering

Taylor-Couette flow-Rayleigh criterion

The exact solution of the Navier-Stokes equations for the flow field between two rotating cylinders is: the velocity $\mathbf{v} = (v_r, v_\varphi, v_z)$ and the pressure $p = p(r)$

$$v_r = v_z = 0 = v_\varphi = v_\varphi(r) = \omega(r)r$$

$$\text{for } \omega(r) = Ar + \frac{B}{r^2},$$

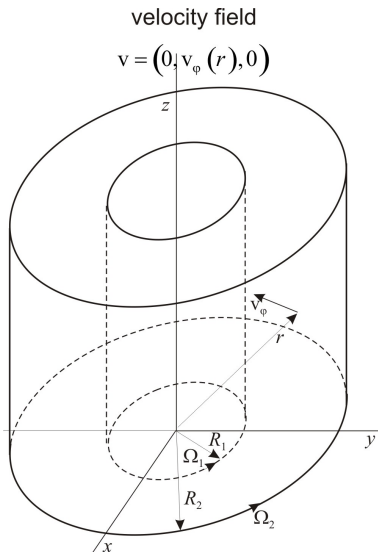
$$A = -\Omega_1 \eta^2 \frac{1 - \mu/\eta^2}{1 - \eta^2},$$

$$B = \Omega_1 \frac{R_1^2(1 - \mu)}{1 - \eta^2}$$

$$\text{for } \mu = \Omega_2/\Omega_1, \eta = R_1/R_2.$$

The radius of the outer cylinder is R_2 and its angular velocity is Ω_2 and for the inner cylinder is R_1, Ω_1 . The nondimensional quantities are $-\infty < \mu = \Omega_2/\Omega_1 < 1$, $0 < \eta = R_1/R_2 < 1$, $0 < \tilde{r} = r/R_2 < 1$. **The Rayleigh Criterion (R.C.) gives**

$$\Omega_2 R_2^2 > \Omega_1 R_1^2$$



The thermodynamic stability condition

$$\frac{\tilde{\pi}}{\mu_{mol}} = \frac{4\Omega_1^2\eta^4(1-\mu)^2}{\tilde{r}^4(1-\eta^2)^2} \geq 0$$

can also be interpreted in the following way

$$\frac{\tilde{\pi}}{\mu_{mol}} = \frac{4\Omega_1^2\eta^4}{\tilde{r}^4} > 0$$

$$\text{for } (1-\mu)^2 = (1-\eta^2)^2$$

$$\text{or } \Omega_1 R_1^2 = \Omega_2 R_2^2 \dots \text{R.C. and}$$

$$\Omega_1 = \Omega_2 \left(2 - \left(\frac{R_1}{R_2} \right)^2 \right)^{-1}$$

The onset of instability is given by the black straight lines $\Omega_1 = 1.292\Omega_2$, for $\Omega_2 > 0$ (right) and for $-\Omega_2 \in (-250, 0)$ by $\Omega_1 =$

$$\frac{-\Omega_2}{\left(\frac{R_1}{R_2}\right)^2 - 2} = 0.815\Omega_2 \text{ (left)}.$$

Considering that for a given ratio of R_1/R_2 , which is always less than 1, the inner cylinder can rotate even faster than the outer one.

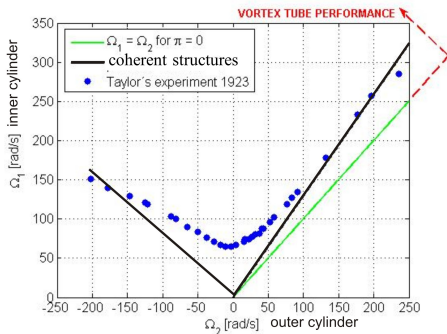


Figure: Onset of coherent structures (instability) for a viscous flow between two rotating cylinders for $\eta = R_1/R_2 = 0.8798$ (Taylor-Couette flow)

Vortex tube experiments

Institute of Aerospace Thermodynamics in Stuttgart shown

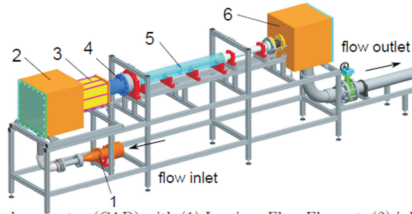


Fig. 1 Experimental apparatus (CAD) with (1) Laminar Flow Element, (2) inlet plenum, (3) mesh heater, (4) swirl generator, (5) swirl tube and (6) outlet plenum [3]

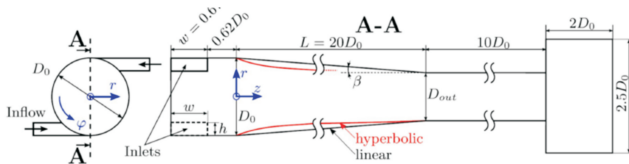
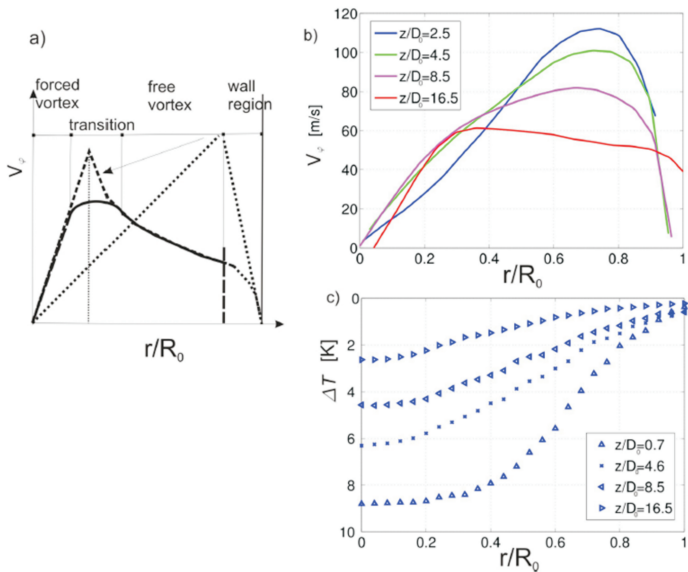


Fig. 2 Convergent swirl chamber [24]

Results of experiments.



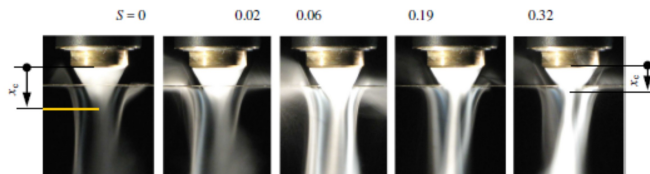
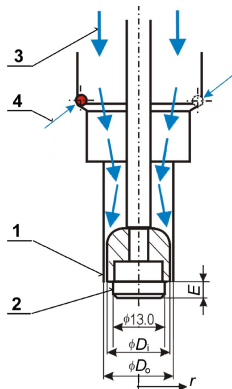
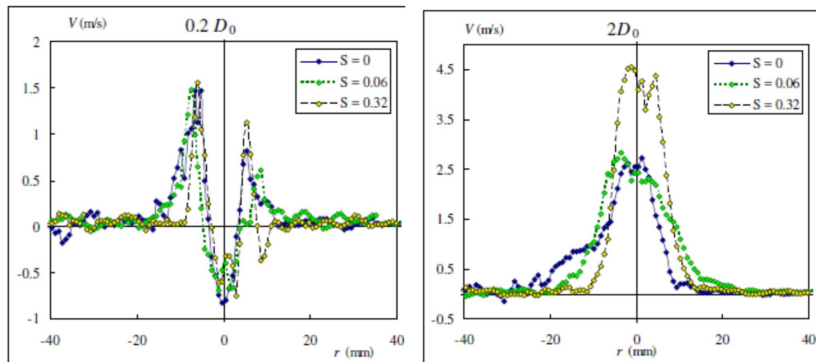


Fig. 2 Flow visualization of the annular jets at $Re = 5000$ for the swirl number S from 0 (without swirling) to 0.32



The annular nozzle and tested configuration. 1: outer nozzle body, 2: nozzle centerbody, 3: axial air flow supply, 4: swirling air flow supply through a pair of tangential ports (diameter 3.1 mm); H : nozzle-to-wall spacing, E : centerbody extension. Dimensions: $D_o = 17.6$ mm, $D_i = 15.85$ mm, and $E = 4.1$ mm.

Annular swirl jet transformation to potential-like vortex

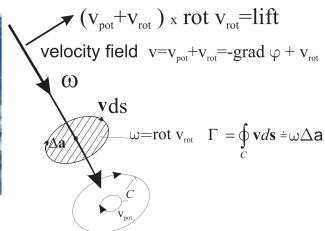


PIV measurement with cross-stream velocity profiles and streamwise velocity component at different distances measured from the bottom part of the nozzle centerbody $(0.2-2)D_0$. The experiments were performed at $Re = 5000$ for five variants of the jets depending on the swirl number S from 0 to 0.32.

Transformation of solid body vortex to potential vortex I

vortex transformation: $\mathbf{v}_{\text{rot}} \longrightarrow \mathbf{v}_{\text{pot}}$

Tropical Cyclone Catarina from the International Space Station
on March 26, 2004



A waterspout near the Florida Keys
(15 miles south of Miami) Tornado over water



Balance laws:

mass

$$\dots \text{div } \mathbf{v} = 0$$

momentum

$$\dots \mathbf{v} \times \text{rot } \mathbf{v} = \text{grad } h_t - T \text{grad } s$$

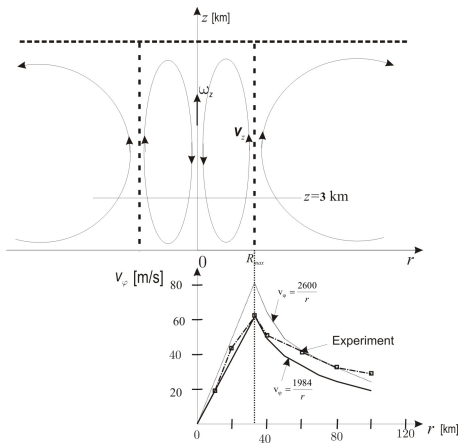
energy redistribution induced by

$$\dots T \text{grad } s \rightarrow \text{grad } h_t$$

entropy change due to condensation

$$\dots T \text{grad } s$$

Transformation of solid body vortex to potential vortex II



Comparison of the qualitative analysis of the cyclone dynamics with the experiment presented in the more detailed analysis (Kerry, A., E.: Tropical Cyclone Energetics and Structure. January (2004)). Values for determining the potential vortex circulation Γ , were taken from experiment, i.e., $\Gamma = v_{\varphi, \text{max}} R_{\text{max}} = 1984$. $\Gamma = 2600$ was used to demonstrate better fitting of the experiment.

- **Vorticity can be induced by dissipation or by volume forces**
- The stability criterion follows from the **Minimum of Total Enthalpy**, or for the steady state from **Maximum of entropy**.
- **Solid body vortex is unstable and transforms into potential vortex**-consequence of the minimum of the total enthalpy or for the steady state the consequence of **Maximum of entropy**.

Our galaxy - the Milky Way



Our galaxy - the Milky Way. Mass $1.29 \cdot 10^9$ Suns, it is 13.6 billion years old and contains 100-400 billion stars. Almost half of the stars were born 4.5 billion years ago. At the core is the supermassive black hole Sagittarius A* with a mass of 4.1 million Suns.

Accretion disk: **Thermodynamic condition of the stability** gives

$$\tilde{\pi} = \mu_{\text{cosmic}} \left[\left(\frac{\partial v_\varphi}{\partial r} \right)^2 + v_\varphi \frac{\partial^2 v_\varphi}{\partial r^2} - \frac{v_\varphi}{r} \frac{\partial v_\varphi}{\partial r} \right] + \frac{\partial \mu_{\text{cosmic}}}{\partial r} \left(v_\varphi \frac{\partial v_\varphi}{\partial r} - \frac{v_\varphi^2}{r} \right) \geq 0$$

The viscosity $\mu_{\text{cosmic}} > 0$ in the flow field $v_\varphi(r)$, ensures the transfer of momentum in the radial direction.

$v_\varphi = \omega r$ then $\tilde{\pi} = 0$ solid body vortex with angular velocity ω is **unstable**

$v_\varphi = \Gamma/r$ then $\tilde{\pi} = 4\mu_{\text{cosmic}} \frac{\Gamma^2}{r^4} - \frac{2\Gamma^2}{r^3} \frac{\partial \mu_{\text{cosmic}}}{\partial r} \geq 0$ potential vortex

Thank you for your attention and I hope this concept offers you some inspiration.



Okay, I'll bring you another wine. I see that gentleman's are true connoisseurs.

Okay, Ich bringe Ihnen noch einen Wein. Ich sehe, Sie, meine Herren, sind wahre Kenner.