Institute of Thermomechanics ASCR

LAGRANGE DESCRIPTION II

Jiří Plešek

Institute of Thermomechanics
Czech Academy of Sciences

Download text: http://www.it.cas.cz/cs/plesek-prednasky/

Contents

- Small strain tensor
- Stretch
- Shear
- Rotation
- Geometrically nonlinear problems

Small strain tensor

Green-Lagrange strain

$$[e] = \frac{1}{2}(\,[F]^T[F] - [I]\,)$$

Small strain tensor

Green-Lagrange strain

$$[e] = \frac{1}{2}(\,[F]^T[F] - [I]\,) = \frac{1}{2}(\,([z] + [I])^T([z] + [I]) - [I]\,)$$

Small strain tensor

Green-Lagrange strain

$$[e] = \frac{1}{2}(\,[F]^T[F] - [I]\,) = \frac{1}{2}(\,([z] + [I])^T([z] + [I]) - [I]\,)$$
$$= \frac{1}{2}(\,[z] + [z]^T + [z]^T[z]\,)$$

Small strain tensor

Green-Lagrange strain

$$[e] = \frac{1}{2}(\,[F]^T[F] - [I]\,) = \frac{1}{2}(\,([z] + [I])^T([z] + [I]) - [I]\,)$$

$$= \frac{1}{2}(\,[z] + [z]^T + [z]^T[z]\,) \simeq \frac{1}{2}(\,[z] + [z]^T\,) \quad \text{for small } [z]$$

Small strain tensor

Green-Lagrange strain

$$[e] = \frac{1}{2}(\,[F]^T[F] - [I]\,) = \frac{1}{2}(\,([z]+[I])^T([z]+[I]) - [I]\,)$$

$$= \frac{1}{2}(\,[z]+[z]^T + [z]^T[z]\,) \simeq \frac{1}{2}(\,[z]+[z]^T\,) \quad \text{for small } [z]$$

Component definition

$$\boxed{\boldsymbol{\epsilon}: \quad [\epsilon] = \frac{1}{2}(\,[z]+[z]^T\,)} \qquad \text{sym}$$

Small strain tensor

Green-Lagrange strain

$$
\begin{aligned}
[e] &= \frac{1}{2}(\,[F]^T[F]-[I]\,) = \frac{1}{2}(\,([z]+[I])^T([z]+[I])-[I]\,) \\
&= \frac{1}{2}(\,[z]+[z]^T+[z]^T[z]\,) \simeq \frac{1}{2}(\,[z]+[z]^T\,) \quad \text{for small } [z]
\end{aligned}
$$

Direct notation

$$
\boxed{\boldsymbol{\epsilon} = \frac{1}{2}(\,\mathbf{z}+\mathbf{z}^T\,)} \qquad \text{sym}
$$

Thus, $\mathbf{e} \to \boldsymbol{\epsilon}$ as $\mathbf{z} \to \mathbf{0}$.

Small strain tensor

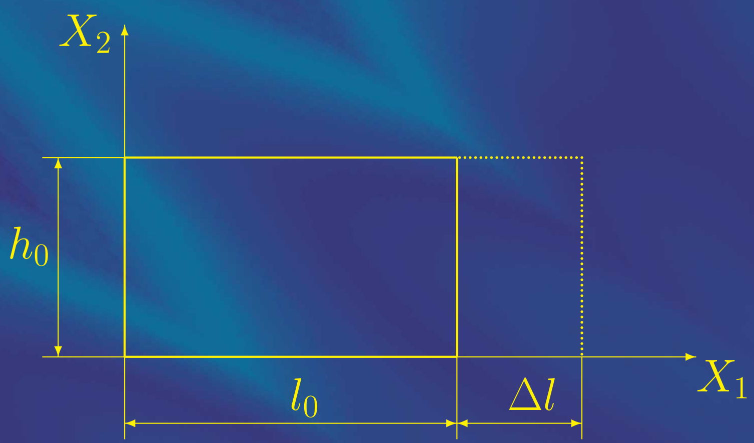
Green-Lagrange strain

$$[e] = \frac{1}{2}(\,[F]^T[F] - [I]\,) = \frac{1}{2}(\,([z]+[I])^T([z]+[I]) - [I]\,)$$
$$= \frac{1}{2}(\,[z] + [z]^T + [z]^T[z]\,) \simeq \frac{1}{2}(\,[z] + [z]^T\,) \quad \text{for small } [z]$$

Index notation

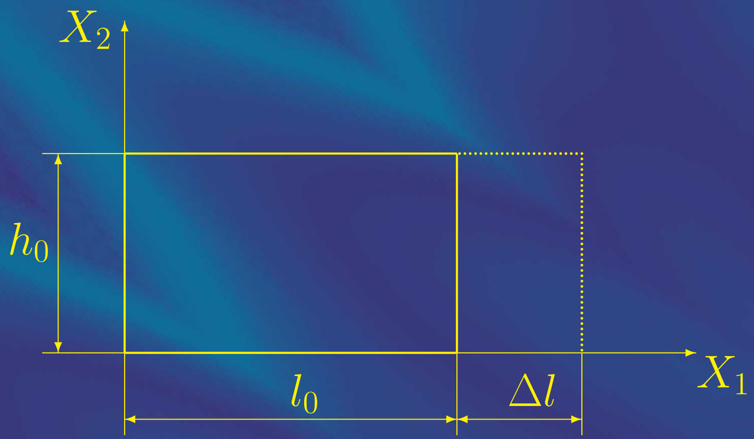
$$\boxed{\epsilon_{ij} = \frac{1}{2}(\,z_{ij} + z_{ji}\,) = \frac{1}{2}\left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i}\right)}$$

Example 2: Stretching (1/3)



$$u_1(X_1, X_2) = \frac{\Delta l}{l_0} X_1$$
$$u_2(X_1, X_2) = 0$$

Example 2: Stretching (1/3)

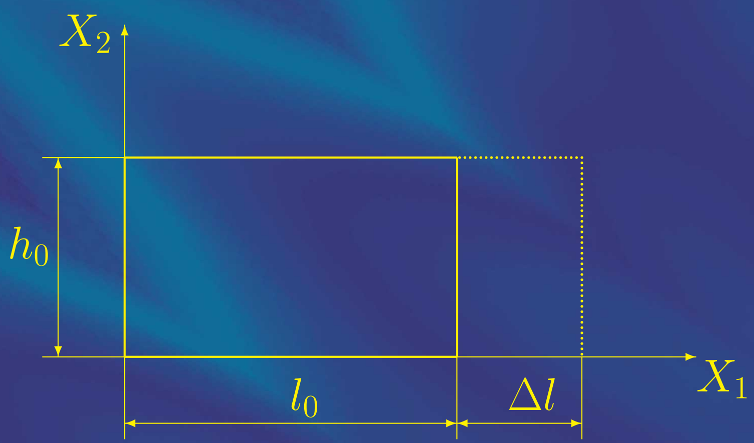


$$u_1(X_1, X_2) = \frac{\Delta l}{l_0} X_1$$
$$u_2(X_1, X_2) = 0$$

Displacement gradient

$$[z] = \begin{bmatrix} \dfrac{\partial u_1}{\partial X_1} & \dfrac{\partial u_1}{\partial X_2} \\ \dfrac{\partial u_2}{\partial X_1} & \dfrac{\partial u_2}{\partial X_2} \end{bmatrix}$$

Example 2: Stretching (1/3)

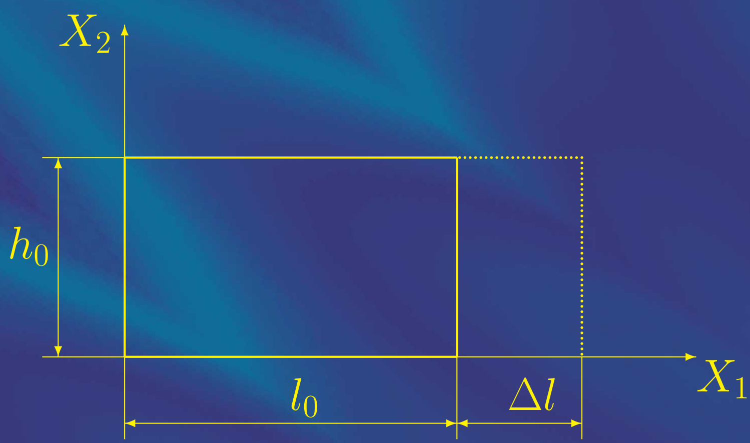


$$u_1(X_1, X_2) = \frac{\Delta l}{l_0} X_1$$
$$u_2(X_1, X_2) = 0$$

Displacement gradient

$$[z] = \begin{bmatrix} \dfrac{\partial u_1}{\partial X_1} & \dfrac{\partial u_1}{\partial X_2} \\ \dfrac{\partial u_2}{\partial X_1} & \dfrac{\partial u_2}{\partial X_2} \end{bmatrix} = \begin{bmatrix} \dfrac{\Delta l}{l_0} & 0 \\ 0 & 0 \end{bmatrix}$$

Example 2: Stretching (1/3)

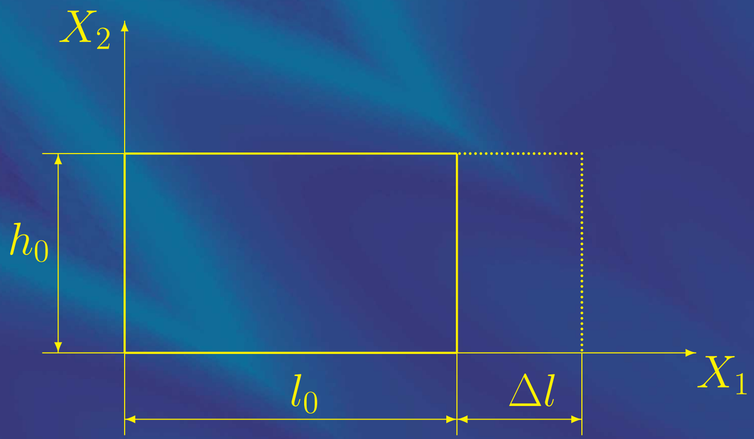


$$u_1(X_1, X_2) = \frac{\Delta l}{l_0} X_1$$
$$u_2(X_1, X_2) = 0$$

Displacement gradient

$$[z] = \begin{bmatrix} \frac{\Delta l}{l_0} & 0 \\ 0 & 0 \end{bmatrix}$$

Example 2: Stretching (1/3)



$$u_1(X_1, X_2) = \frac{\Delta l}{l_0} X_1$$
$$u_2(X_1, X_2) = 0$$

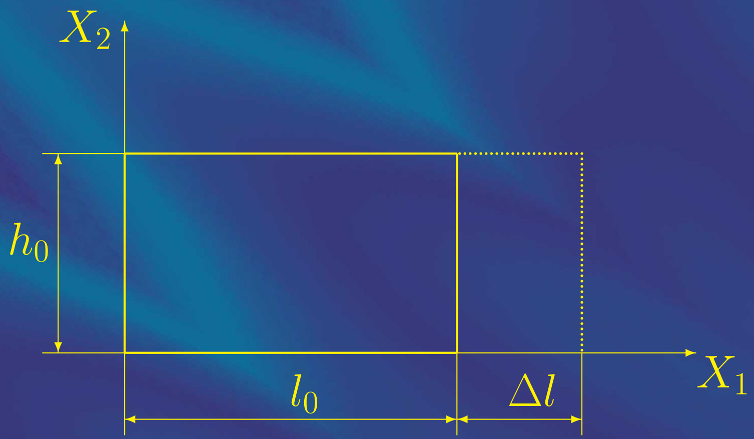
Displacement gradient

$$[z] = \begin{bmatrix} \frac{\Delta l}{l_0} & 0 \\ 0 & 0 \end{bmatrix}$$

Deformation gradient

$$[F] = [z] + [I]$$

Example 2: Stretching (1/3)



$$u_1(X_1, X_2) = \frac{\Delta l}{l_0} X_1$$
$$u_2(X_1, X_2) = 0$$

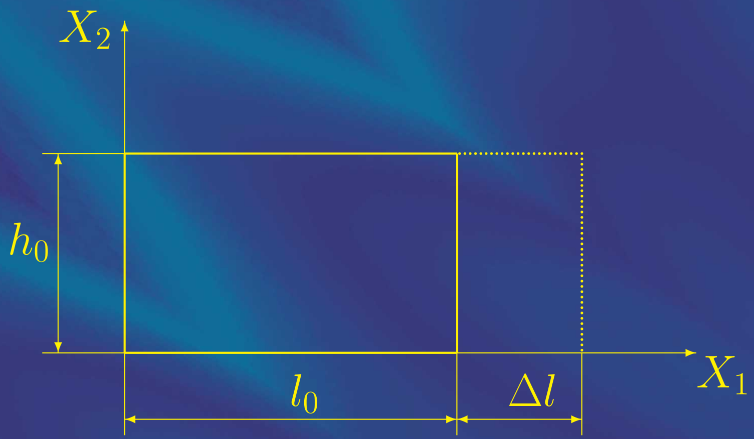
Displacement gradient

$$[z] = \begin{bmatrix} \frac{\Delta l}{l_0} & 0 \\ 0 & 0 \end{bmatrix}$$

Deformation gradient

$$[F] = \begin{bmatrix} 1 + \frac{\Delta l}{l_0} & 0 \\ 0 & 1 \end{bmatrix}$$

Example 2: Stretching (1/3)



$$u_1(X_1, X_2) = \frac{\Delta l}{l_0} X_1$$
$$u_2(X_1, X_2) = 0$$

Displacement gradient

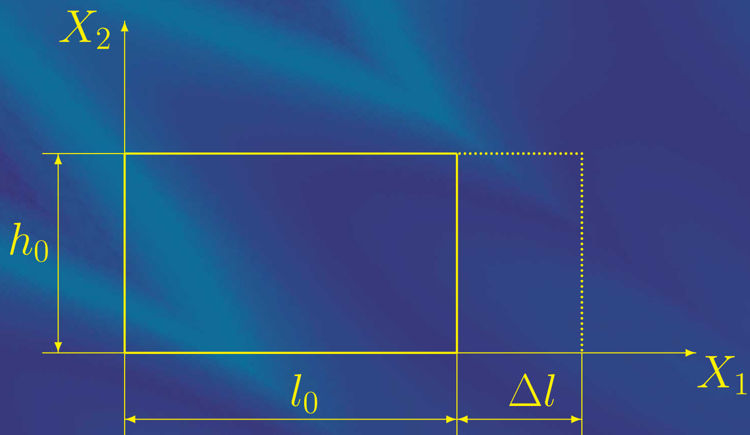
$$[z] = \begin{bmatrix} \frac{\Delta l}{l_0} & 0 \\ 0 & 0 \end{bmatrix}$$

Deformation gradient

$$[F] = \begin{bmatrix} 1 + \frac{\Delta l}{l_0} & 0 \\ 0 & 1 \end{bmatrix}$$

$$J = 1 + \frac{\Delta l}{l_0} > 0$$

Example 2: Stretching (1/3)



$$u_1(X_1, X_2) = \frac{\Delta l}{l_0} X_1$$
$$u_2(X_1, X_2) = 0$$

Displacement gradient

$$[z] = \begin{bmatrix} \frac{\Delta l}{l_0} & 0 \\ 0 & 0 \end{bmatrix}$$

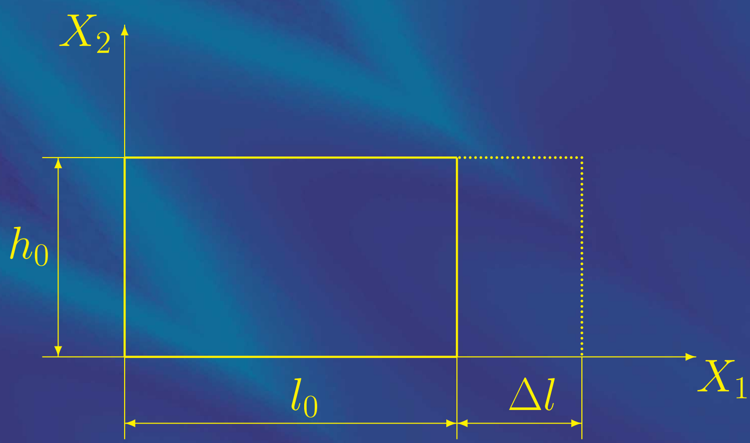
Deformation gradient

$$[F] = \begin{bmatrix} 1 + \frac{\Delta l}{l_0} & 0 \\ 0 & 1 \end{bmatrix}$$

$$J = 1 + \frac{\Delta l}{l_0} > 0$$

$\Delta l > -l_0$ (regularity)

Example 2: Stretching (1/3)



$$u_1(X_1, X_2) = \frac{\Delta l}{l_0} X_1$$
$$u_2(X_1, X_2) = 0$$

Displacement gradient

$$[z] = \begin{bmatrix} \frac{\Delta l}{l_0} & 0 \\ 0 & 0 \end{bmatrix}$$

Deformation gradient

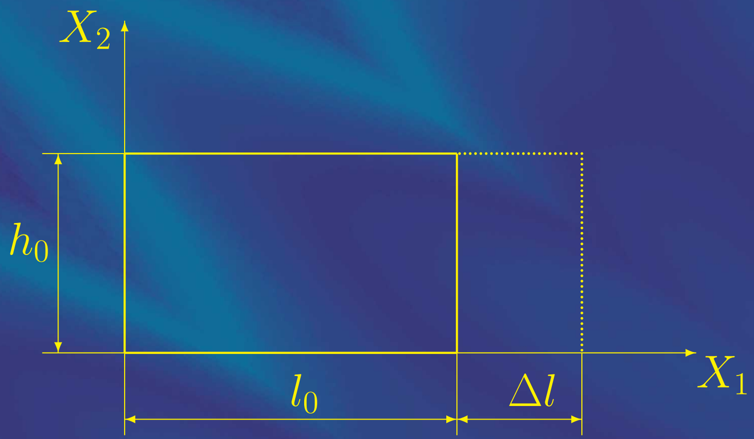
$$[F] = \begin{bmatrix} 1 + \frac{\Delta l}{l_0} & 0 \\ 0 & 1 \end{bmatrix}$$

$$J = 1 + \frac{\Delta l}{l_0} > 0$$

$\Delta l > -l_0$ (regularity)

$$\frac{V_t}{V_0} = \frac{h_0(l_0 + \Delta l)}{h_0 l_0} = J$$

Example 2: Stretching (1/3)



$$u_1(X_1, X_2) = \frac{\Delta l}{l_0} X_1$$
$$u_2(X_1, X_2) = 0$$

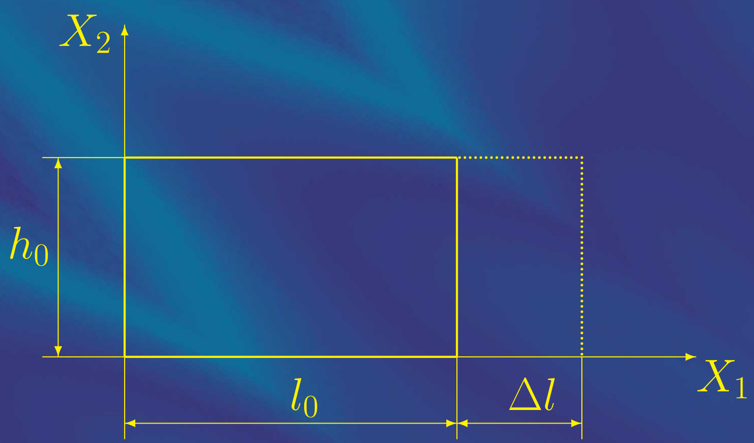
Displacement gradient

$$[z] = \begin{bmatrix} \frac{\Delta l}{l_0} & 0 \\ 0 & 0 \end{bmatrix}$$

Small strain tensor

$$[\epsilon] = \frac{1}{2}\left([z] + [z]^T \right)$$

Example 2: Stretching (1/3)



$$u_1(X_1, X_2) = \frac{\Delta l}{l_0} X_1$$
$$u_2(X_1, X_2) = 0$$

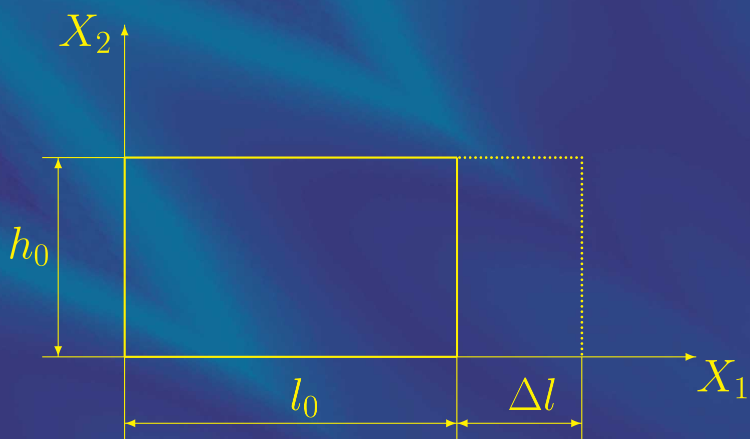
Displacement gradient

$$[z] = \begin{bmatrix} \frac{\Delta l}{l_0} & 0 \\ 0 & 0 \end{bmatrix}$$

Small strain tensor

$$[\epsilon] = \begin{bmatrix} \frac{\Delta l}{l_0} & 0 \\ 0 & 0 \end{bmatrix}$$

Example 2: Stretching (1/3)



$$u_1(X_1, X_2) = \frac{\Delta l}{l_0} X_1$$
$$u_2(X_1, X_2) = 0$$

Displacement gradient

$$[z] = \begin{bmatrix} \frac{\Delta l}{l_0} & 0 \\ 0 & 0 \end{bmatrix}$$

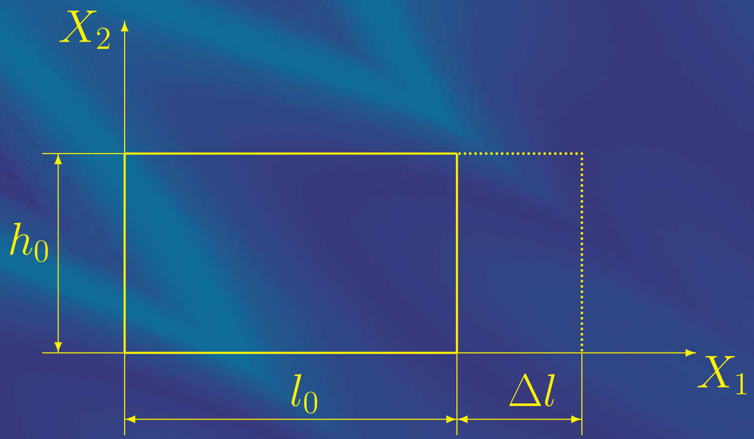
Small strain tensor

$$[\epsilon] = \begin{bmatrix} \frac{\Delta l}{l_0} & 0 \\ 0 & 0 \end{bmatrix}$$

GL strain tensor

$$[e] = [\epsilon] + \frac{1}{2}[z]^T[z]$$

Example 2: Stretching (1/3)



$$u_1(X_1, X_2) = \frac{\Delta l}{l_0} X_1$$
$$u_2(X_1, X_2) = 0$$

Displacement gradient

$$[z] = \begin{bmatrix} \frac{\Delta l}{l_0} & 0 \\ 0 & 0 \end{bmatrix}$$

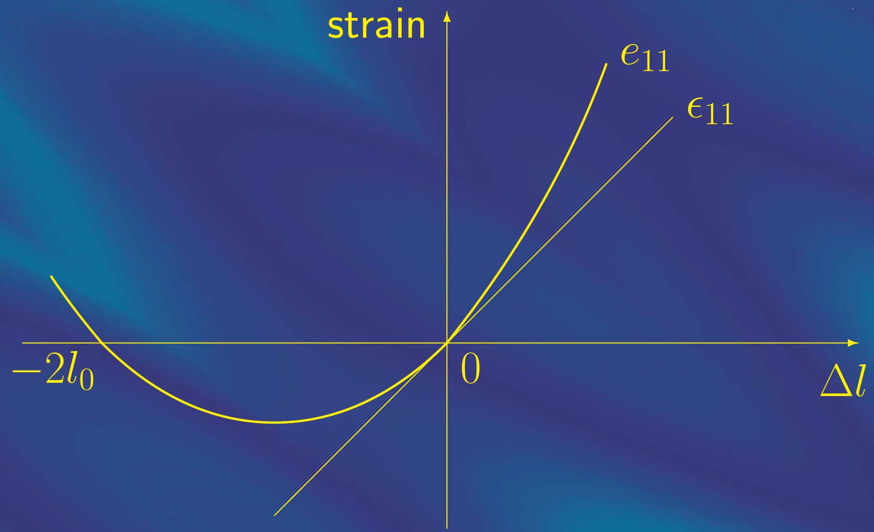
Small strain tensor

$$[\epsilon] = \begin{bmatrix} \frac{\Delta l}{l_0} & 0 \\ 0 & 0 \end{bmatrix}$$

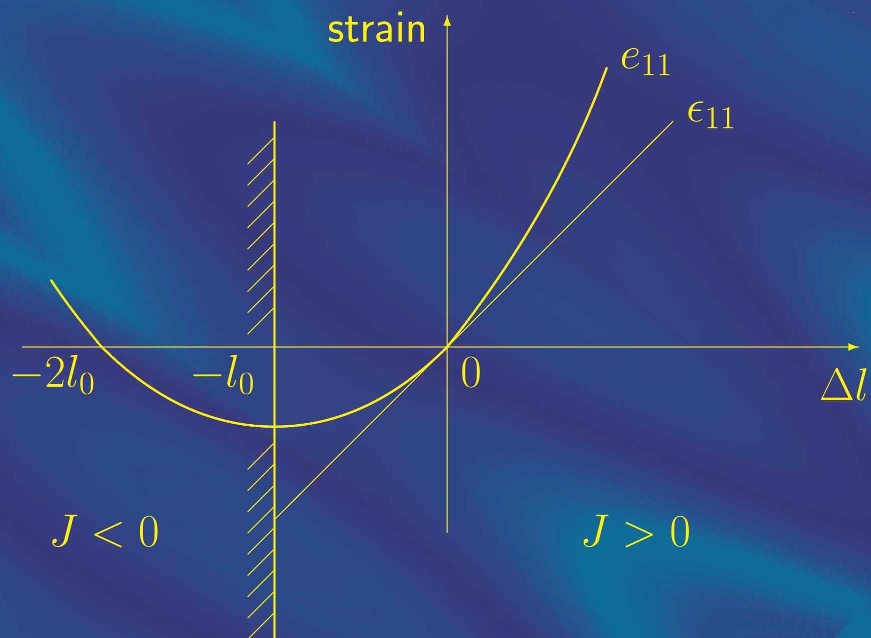
GL strain tensor

$$[e] = \begin{bmatrix} \frac{\Delta l}{l_0} + \frac{1}{2}\left(\frac{\Delta l}{l_0}\right)^2 & 0 \\ 0 & 0 \end{bmatrix}$$

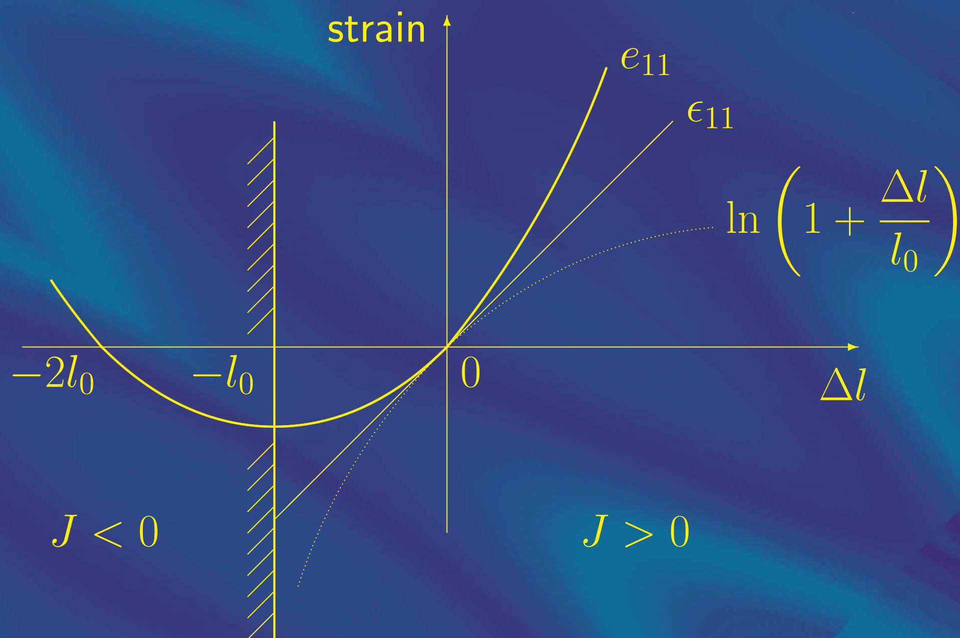
Example 2: Stretching (2/3)



Example 2: Stretching (2/3)



Example 2: Stretching (2/3)



Example 2: Stretching (3/3)

Conclusion for stretching mode (moral)

- The small strain tensor may well be used even for large deformation. Diagonal components represent relative elongations regardless of the strain magnitude.

Example 2: Stretching (3/3)

Conclusion for stretching mode (moral)

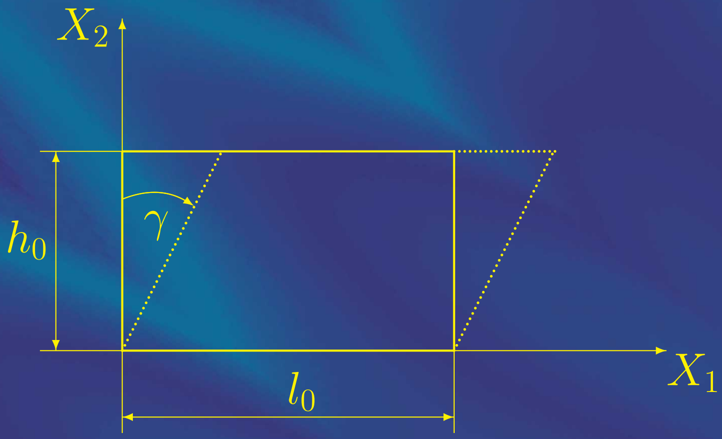
- The small strain tensor may well be used even for large deformation. Diagonal components represent relative elongations regardless of the strain magnitude.
- The Green-Lagrange strain tensor is by no means better (if not worse) measure of stretching than the small strain tensor.

Example 2: Stretching (3/3)

Conclusion for stretching mode (moral)

- The small strain tensor may well be used even for large deformation. Diagonal components represent relative elongations regardless of the strain magnitude.
- The Green-Lagrange strain tensor is by no means better (if not worse) measure of stretching than the small strain tensor.
- However, at the end of the day, all 'normal' strain tensors are equivalent as the change of one for another only means argument substitution in the free energy function. A choice of a proper strain tensor is, thus, merely a matter of convenience.

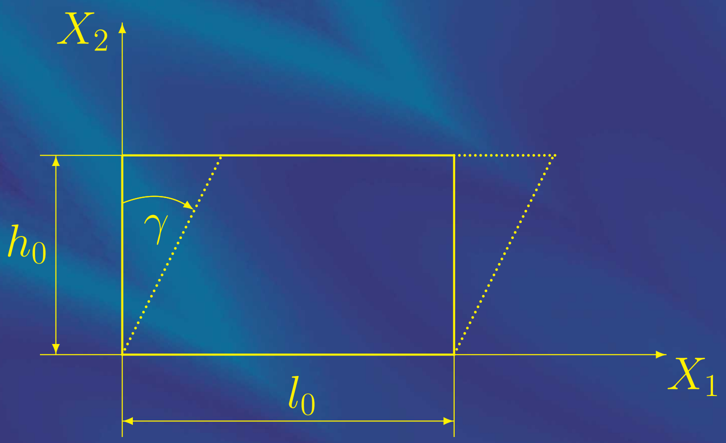
Example 3: Shearing (1/2)



$$u_1(X_1, X_2) = X_2 \tan\gamma$$

$$u_2(X_1, X_2) = 0$$

Example 3: Shearing (1/2)

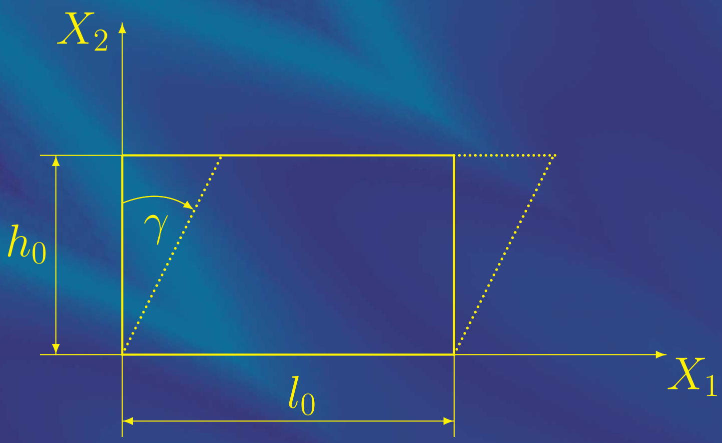


$$u_1(X_1, X_2) = X_2 \tan\gamma$$
$$u_2(X_1, X_2) = 0$$

Displacement gradient

$$[z] = \begin{bmatrix} 0 & \tan\gamma \\ 0 & 0 \end{bmatrix}$$

Example 3: Shearing (1/2)



$$u_1(X_1, X_2) = X_2 \tan\gamma$$
$$u_2(X_1, X_2) = 0$$

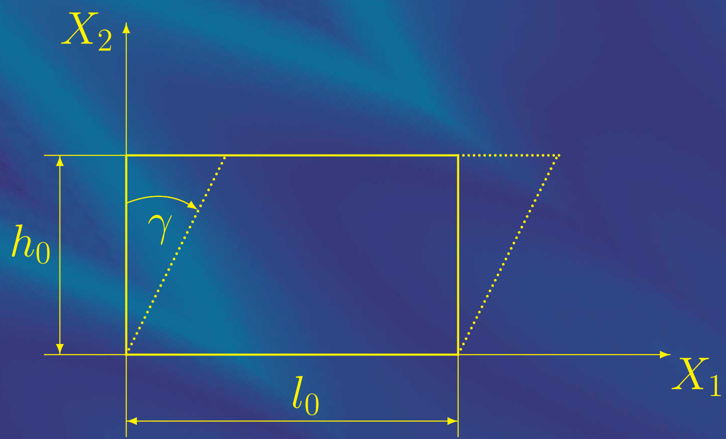
Displacement gradient

$$[z] = \begin{bmatrix} 0 & \tan\gamma \\ 0 & 0 \end{bmatrix}$$

Deformation gradient

$$[F] = \begin{bmatrix} 1 & \tan\gamma \\ 0 & 1 \end{bmatrix}$$

Example 3: Shearing (1/2)



$$u_1(X_1, X_2) = X_2 \tan\gamma$$
$$u_2(X_1, X_2) = 0$$

Displacement gradient

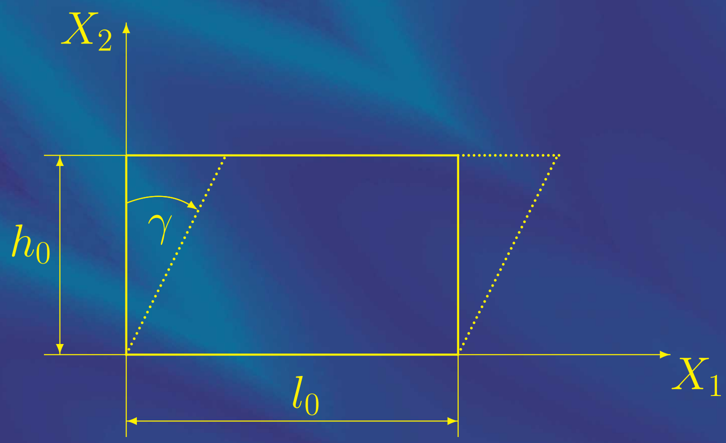
$$[z] = \begin{bmatrix} 0 & \tan\gamma \\ 0 & 0 \end{bmatrix}$$

Deformation gradient

$$[F] = \begin{bmatrix} 1 & \tan\gamma \\ 0 & 1 \end{bmatrix}$$

$J = 1$ (isochoric)

Example 3: Shearing (1/2)



$$u_1(X_1, X_2) = X_2 \tan\gamma$$
$$u_2(X_1, X_2) = 0$$

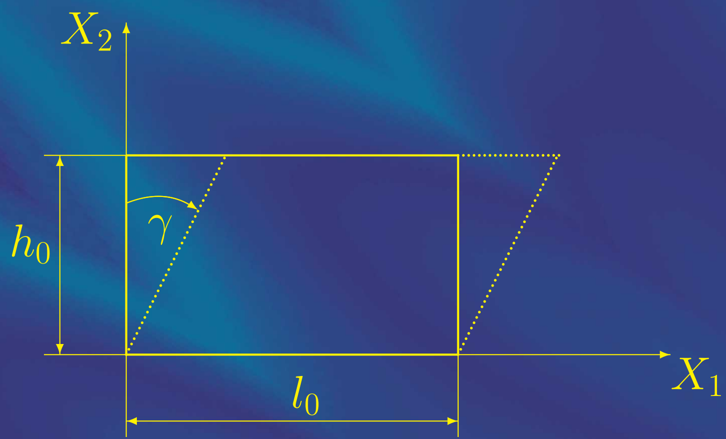
Displacement gradient

$$[z] = \begin{bmatrix} 0 & \tan\gamma \\ 0 & 0 \end{bmatrix}$$

Small strain tensor

$$[\epsilon] = \frac{1}{2}\begin{bmatrix} 0 & \tan\gamma \\ \tan\gamma & 0 \end{bmatrix}$$

Example 3: Shearing (1/2)



$$u_1(X_1, X_2) = X_2 \tan\gamma$$
$$u_2(X_1, X_2) = 0$$

Displacement gradient

$$[z] = \begin{bmatrix} 0 & \tan\gamma \\ 0 & 0 \end{bmatrix}$$

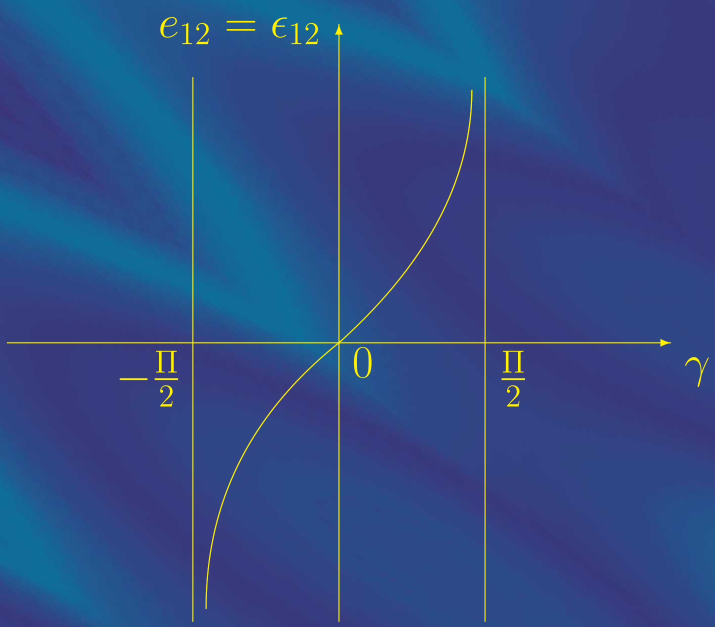
Small strain tensor

$$[\epsilon] = \frac{1}{2}\begin{bmatrix} 0 & \tan\gamma \\ \tan\gamma & 0 \end{bmatrix}$$

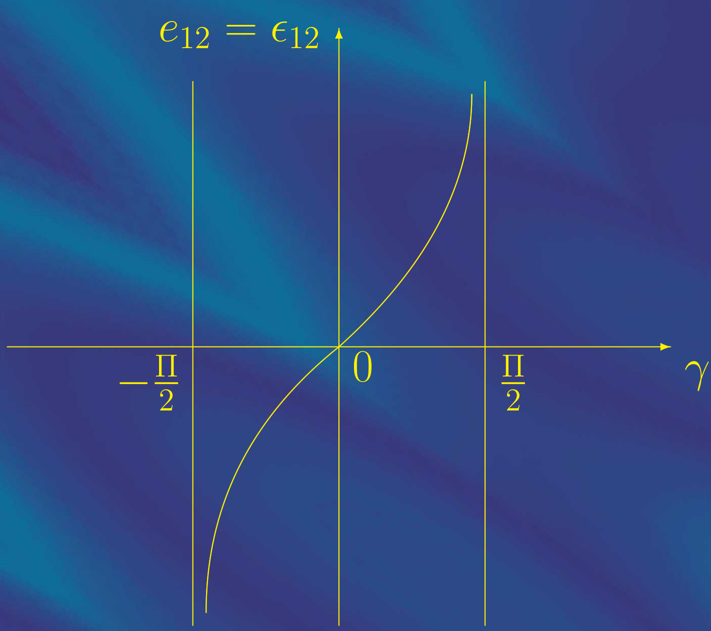
GL strain tensor

$$[\epsilon] = \frac{1}{2}\begin{bmatrix} 0 & \tan\gamma \\ \tan\gamma & \tan^2\gamma \end{bmatrix}$$

Example 3: Shearing (2/2)

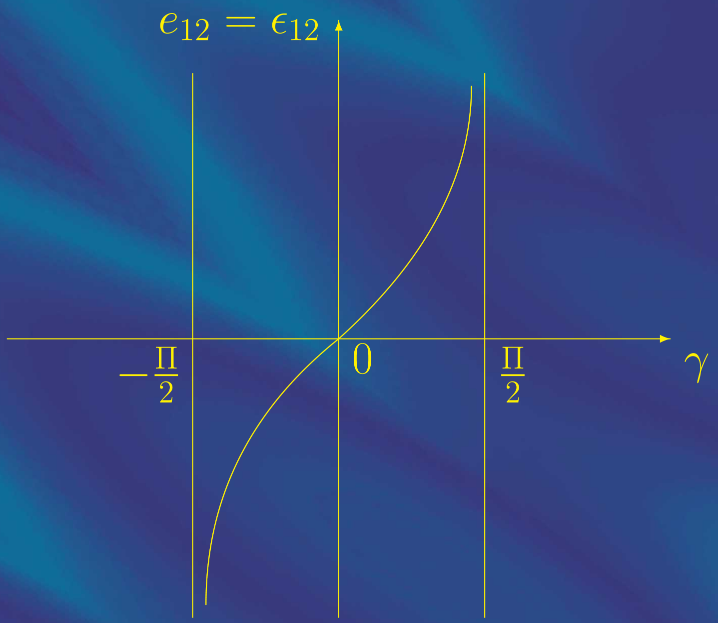


Example 3: Shearing (2/2)



remark: for $\gamma \to 0$, $\tan\gamma \simeq \gamma$

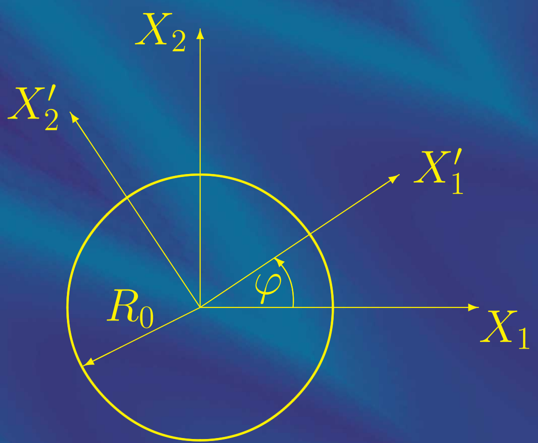
Example 3: Shearing (2/2)



- The small strain tensor contains tangents of shear angles, $\tan\gamma$, but not the shear angles, γ, themselves. This, for large distortion, is even better measure of shearing.
- The Green-Langrange strain tensor offers the exact same description. So far, no advantage of GL has been demonstrated.

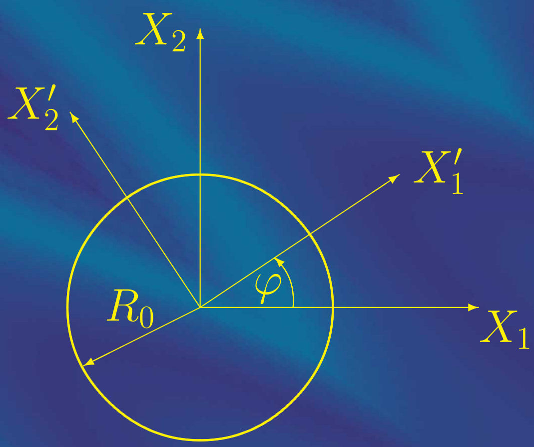
remark: for $\gamma \to 0$, $\tan\gamma \simeq \gamma$

Example 4: Rotation



$$u_1(X_1, X_2) = X_1(\cos\varphi - 1) - X_2 \sin\varphi$$
$$u_2(X_1, X_2) = X_1 \sin\varphi + X_2(\cos\varphi - 1)$$

Example 4: Rotation



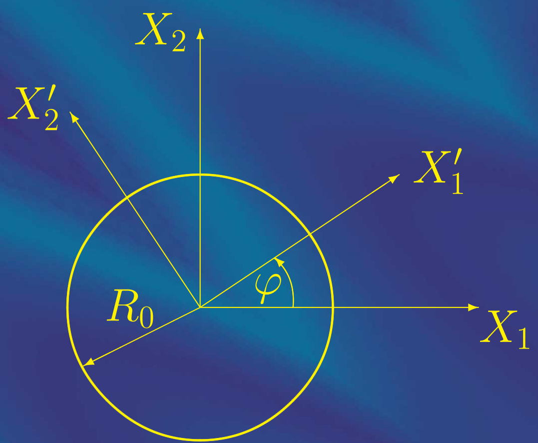
$$u_1(X_1, X_2) = X_1(\cos\varphi - 1) - X_2 \sin\varphi$$

$$u_2(X_1, X_2) = X_1 \sin\varphi + X_2(\cos\varphi - 1)$$

Transformation matrix

$$[A] = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}$$

Example 4: Rotation



$$u_1(X_1, X_2) = X_1(\cos\varphi - 1) - X_2 \sin\varphi$$

$$u_2(X_1, X_2) = X_1 \sin\varphi + X_2(\cos\varphi - 1)$$

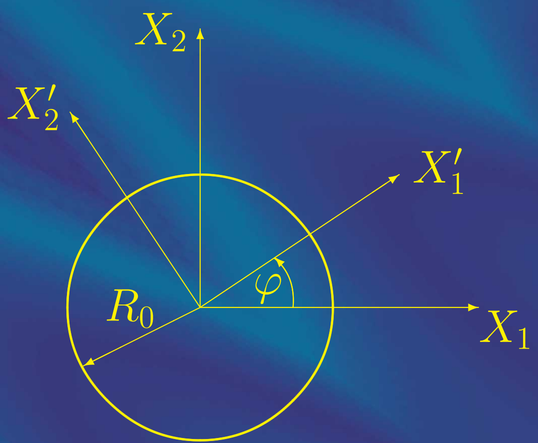
Transformation matrix

$$[A] = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}$$

Rotation condition

$$\{x'\} \equiv \{X\}$$

Example 4: Rotation



$$u_1(X_1, X_2) = X_1(\cos\varphi - 1) - X_2 \sin\varphi$$

$$u_2(X_1, X_2) = X_1 \sin\varphi + X_2(\cos\varphi - 1)$$

Transformation matrix

$$[A] = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}$$

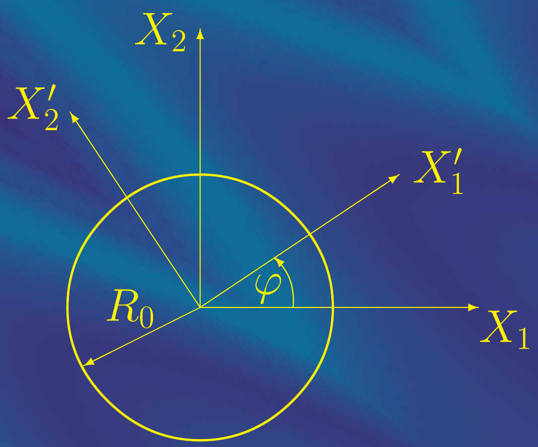
Rotation condition

$$\{x'\} \equiv \{X\}$$

Displacement field

$$\{u\} = \{x\} - \{X\}$$

Example 4: Rotation



$$u_1(X_1, X_2) = X_1(\cos\varphi - 1) - X_2 \sin\varphi$$

$$u_2(X_1, X_2) = X_1 \sin\varphi + X_2(\cos\varphi - 1)$$

Transformation matrix

$$[A] = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}$$

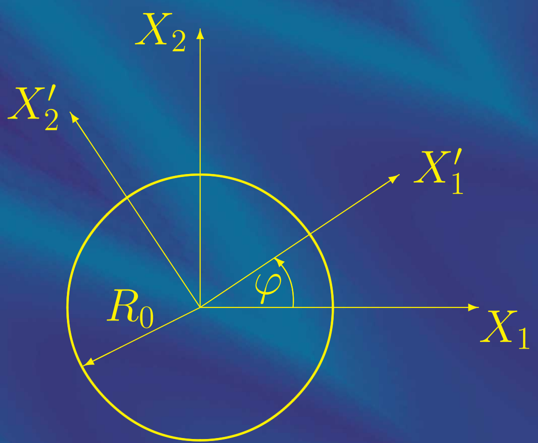
Rotation condition

$$\{x'\} \equiv \{X\}$$

Displacement field

$$\begin{aligned} \{u\} &= \{x\} - \{X\} \\ &= [A]^T\{x'\} - \{X\} \end{aligned}$$

Example 4: Rotation



$$u_1(X_1, X_2) = X_1(\cos\varphi - 1) - X_2 \sin\varphi$$

$$u_2(X_1, X_2) = X_1 \sin\varphi + X_2(\cos\varphi - 1)$$

Transformation matrix

$$[A] = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}$$

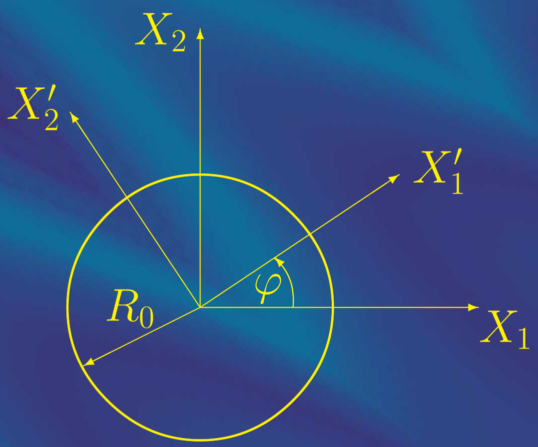
Rotation condition

$$\{x'\} \equiv \{X\}$$

Displacement field

$$\begin{aligned} \{u\} &= \{x\} - \{X\} \\ &= [A]^T\{x'\} - \{X\} \\ &= [A]^T\{X\} - \{X\} \end{aligned}$$

Example 4: Rotation



$$u_1(X_1, X_2) = X_1(\cos\varphi - 1) - X_2 \sin\varphi$$

$$u_2(X_1, X_2) = X_1 \sin\varphi + X_2(\cos\varphi - 1)$$

Transformation matrix

$$[A] = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}$$

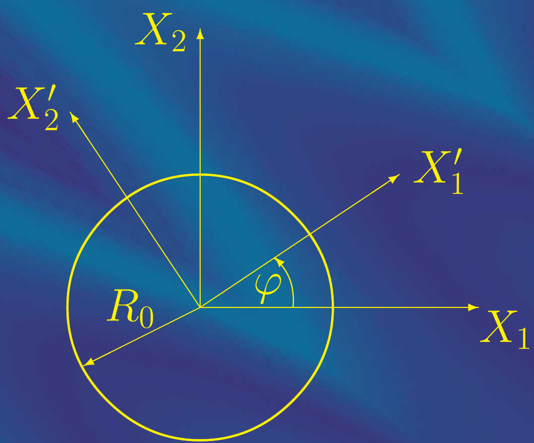
Rotation condition

$$\{x'\} \equiv \{X\}$$

Displacement field

$$\begin{aligned} \{u\} &= \{x\} - \{X\} \\ &= [A]^T\{x'\} - \{X\} \\ &= \left([A]^T - [I] \right)\{X\} \end{aligned}$$

Example 4: Rotation



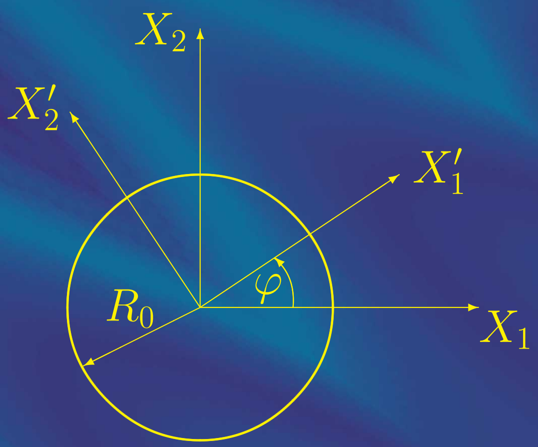
$$u_1(X_1, X_2) = X_1(\cos\varphi - 1) - X_2 \sin\varphi$$

$$u_2(X_1, X_2) = X_1 \sin\varphi + X_2(\cos\varphi - 1)$$

Transformation matrix

$$[A] = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}$$

Example 4: Rotation



$$u_1(X_1, X_2) = X_1(\cos\varphi - 1) - X_2 \sin\varphi$$
$$u_2(X_1, X_2) = X_1 \sin\varphi + X_2(\cos\varphi - 1)$$

Transformation matrix

$$[A] = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}$$

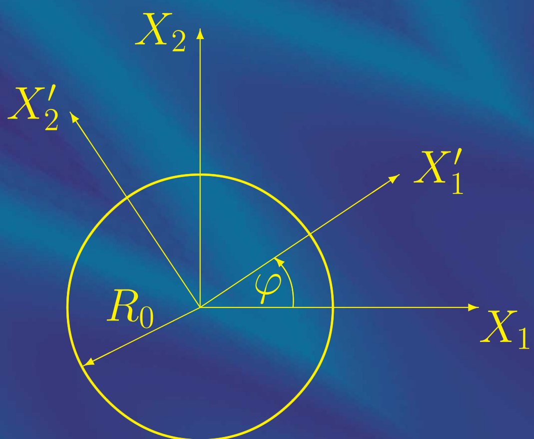
Displacement gradient

$$[z] = \begin{bmatrix} \cos\varphi - 1 & -\sin\varphi \\ \sin\varphi & \cos\varphi - 1 \end{bmatrix}$$

Deformation gradient

$$[F] = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}$$

Example 4: Rotation



$$u_1(X_1, X_2) = X_1(\cos\varphi - 1) - X_2 \sin\varphi$$
$$u_2(X_1, X_2) = X_1 \sin\varphi + X_2(\cos\varphi - 1)$$

Transformation matrix

$$[A] = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}$$

Displacement gradient

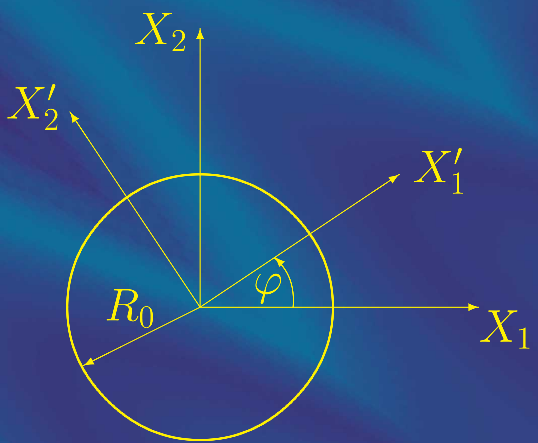
$$[z] = \begin{bmatrix} \cos\varphi - 1 & -\sin\varphi \\ \sin\varphi & \cos\varphi - 1 \end{bmatrix}$$

Deformation gradient

$$[F] = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}$$

$[F] = [A]^T$ (orthonormal), $J = \cos^2\varphi + \sin^2\varphi = 1$ (isochoric)

Example 4: Rotation

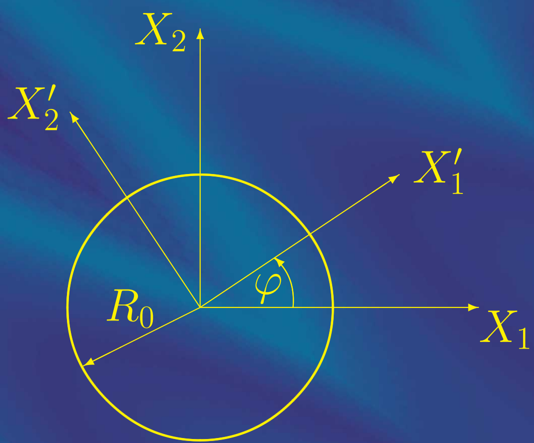


$$u_1(X_1, X_2) = X_1(\cos\varphi - 1) - X_2 \sin\varphi$$
$$u_2(X_1, X_2) = X_1 \sin\varphi + X_2(\cos\varphi - 1)$$

Small strain tensor

$$[\epsilon] = \begin{bmatrix} \cos\varphi - 1 & 0 \\ 0 & \cos\varphi - 1 \end{bmatrix}$$

Example 4: Rotation



$$u_1(X_1, X_2) = X_1(\cos\varphi - 1) - X_2 \sin\varphi$$

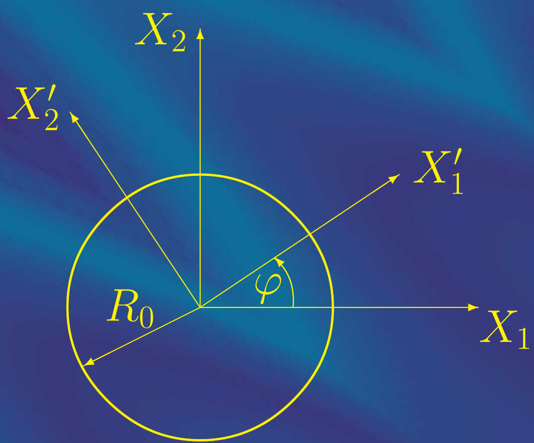
$$u_2(X_1, X_2) = X_1 \sin\varphi + X_2(\cos\varphi - 1)$$

Small strain tensor

$$[\epsilon] = \begin{bmatrix} \cos\varphi - 1 & 0 \\ 0 & \cos\varphi - 1 \end{bmatrix}$$

- Not invariant under rigid body rotation.
- For small rotation, $\cos\varphi \simeq 1$ as $\varphi \to 0$.

Example 4: Rotation



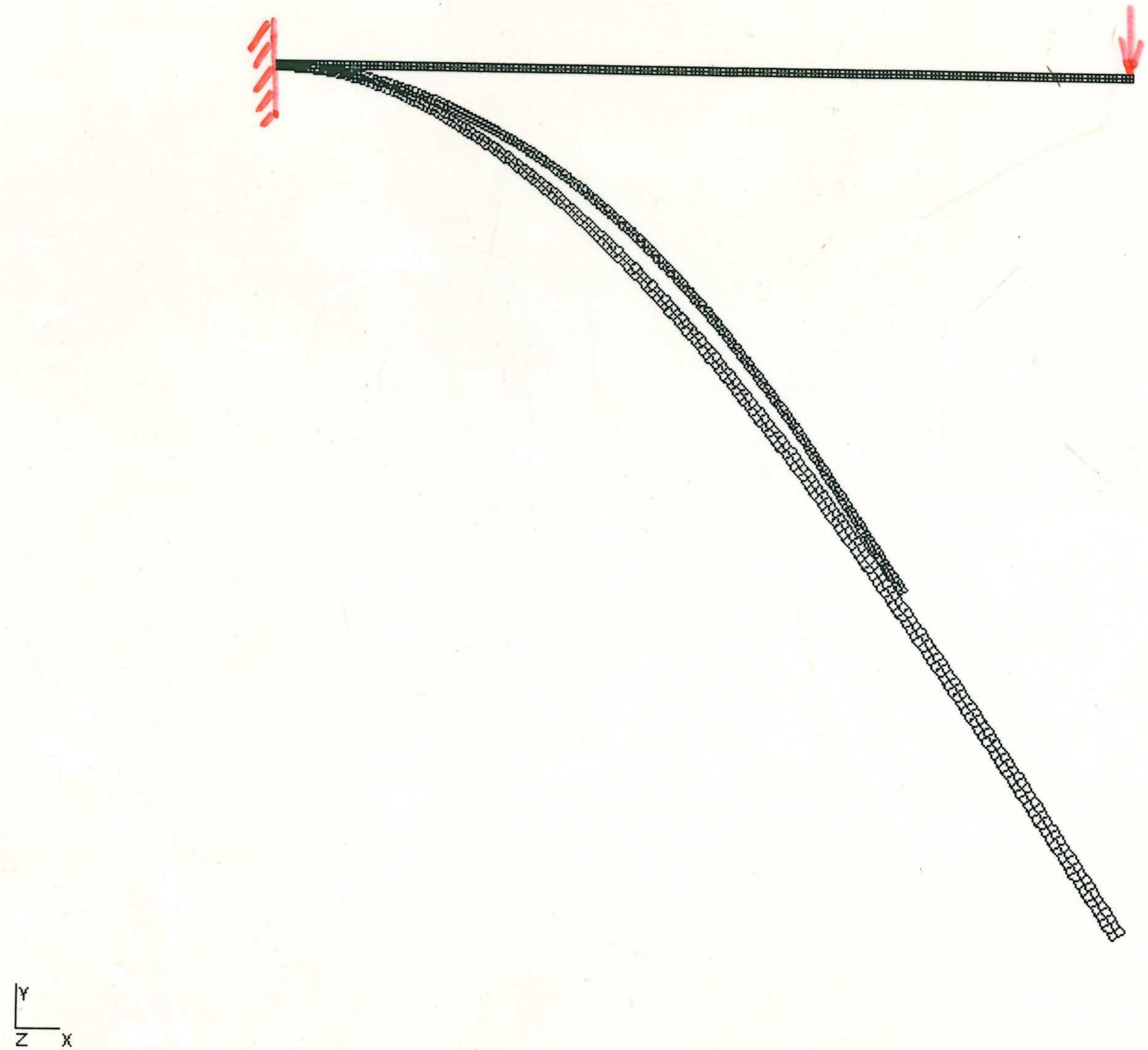
$$u_1(X_1, X_2) = X_1(\cos\varphi - 1) - X_2 \sin\varphi$$

$$u_2(X_1, X_2) = X_1 \sin\varphi + X_2(\cos\varphi - 1)$$

Small strain tensor

$$[\epsilon] = \begin{bmatrix} \cos\varphi - 1 & 0 \\ 0 & \cos\varphi - 1 \end{bmatrix}$$

- Not invariant under rigid body rotation.
- For small rotation, $\cos\varphi \simeq 1$ as $\varphi \to 0$.
- Example: $\varphi = 1°$, $\epsilon_{11} = -1.5 \times 10^{-4}$
  $\sigma_{11} \simeq E\epsilon_{11} = -30$ MPa.


Y
Z X

Conclusions

- All possible states were covered. The three modes combine into any pattern of the deformation gradient. Shrinking the solution domain, l_0, $h_0 \to 0$ or, equivalently, setting $l_0 = \mathrm{d}X_1$ and $h_0 = \mathrm{d}X_2$, accounts for the inclusion of nonlinear displacement fields.

Conclusions

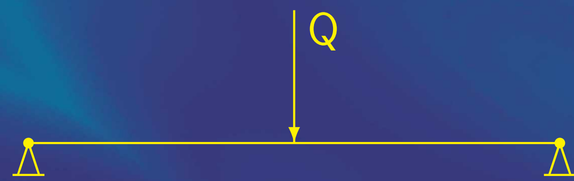
- All possible states were covered. The three modes combine into any pattern of the deformation gradient. Shrinking the solution domain, l_0, $h_0 \to 0$ or, equivalently, setting $l_0 = \mathrm{d}X_1$ and $h_0 = \mathrm{d}X_2$, accounts for the inclusion of nonlinear displacement fields.
- Considering straining modes, i.e. stretching and shearing, no advantage of the Green-Lagrange strain should be claimed over the small strain tensor. Problems are posed by rotation. Thus, the use of small strain approach is limited to problems with small rotation but possibly large deformation.

Conclusions

- All possible states were covered. The three modes combine into any pattern of the deformation gradient. Shrinking the solution domain, l_0, $h_0 \rightarrow 0$ or, equivalently, setting $l_0 = \mathrm{d}X_1$ and $h_0 = \mathrm{d}X_2$, accounts for the inclusion of nonlinear displacement fields.
- Considering straining modes, i.e. stretching and shearing, no advantage of the Green-Lagrange strain should be claimed over the small strain tensor. Problems are posed by rotation. Thus, the use of small strain approach is limited to problems with small rotation but possibly large deformation.
- It follows, that the knowledge of strain compoments alone does not guarantee the correct application of the small strain theory. In most cases, though, it is the displacement field that is damaged, not stresses. A class of problems requiring the employment of GL-tensor is said to be geometrically nonlinear.

Geometrically nonlinear problems (1/2)

A prototype 'fly' problem

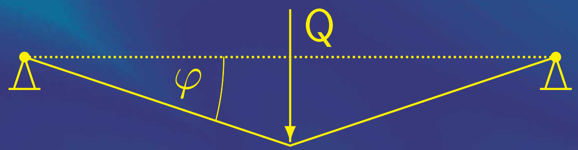


(initial stress $= S_0$)

Geometrically nonlinear problems (1/2)

A prototype 'fly' problem

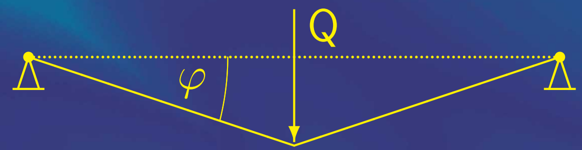
(initial stress $= S_0$)



Geometrically nonlinear problems (1/2)

A prototype 'fly' problem

(initial stress $= S_0$)



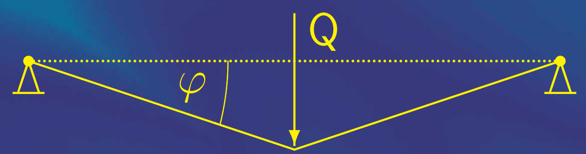
Equilibrium equation

$$2S \sin\varphi = Q$$

Geometrically nonlinear problems (1/2)

A prototype 'fly' problem

(initial stress $= S_0$)



Equilibrium equation

$$2S \sin\varphi = Q$$

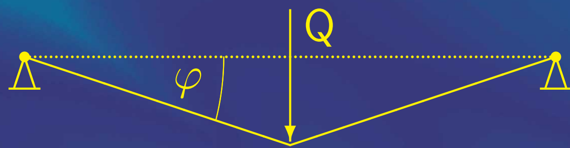
Linearization

$$S \simeq S_0, \quad \sin\varphi \simeq \tan\varphi = 2v/l_0$$

Geometrically nonlinear problems (1/2)

A prototype 'fly' problem

(initial stress $= S_0$)



Equilibrium equation

$$2S\sin\varphi = Q$$

Linearization

$$S \simeq S_0, \quad \sin\varphi \simeq \tan\varphi = 2v/l_0$$

Solution

$$v = \frac{Ql_0}{4S_0}$$

Despite the solution is linear we speak about 'geometric nonlinearity' as the equilibrium equation was written for a deformed state and linearized only later.

Geometrically nonlinear problems (2/2)

Small stresses, strains and even rotations yet the latter must be taken into account using a proper description, usually via the Green-Lagrange strain tensor.

Geometrically nonlinear problems (2/2)

Small stresses, strains and even rotations yet the latter must be taken into account using a proper description, usually via the Green-Lagrange strain tensor.

- Cables, membranes, plates, shells.
- Stability of thin walled structures.
- Vibration of pre-stressed structures (turbine blades).
- FEM – initial stress matrix.

Geometrically nonlinear problems (2/2)

Small stresses, strains and even rotations yet the latter must be taken into account using a proper description, usually via the Green-Lagrange strain tensor.

- Cables, membranes, plates, shells.
- Stability of thin walled structures.
- Vibration of pre-stressed structures (turbine blades).
- FEM – initial stress matrix.

Classification

- ϵ + Hooke   linear elasticity
- ϵ + plast.   materially nonlinear problems
- e + Hooke   geometrically nonlinear problems