NUMERICAL APPROACH OF THE FLOW BETWEEN ROTATING COAXIAL CONES

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Abstract

This paper presents a numerical study of Taylor-Couette flows between two coaxial cones when the inner cone is rotating rotates with a constant angular velocity and the outer one is at rest. The cones have the same apex angle $\phi = 12$ degrees resulting in a constant gap width so that $\delta = d/R_{1\text{max}} = 0.12$. Our primary interests are focusing on analysis of laminar-turbulent regime to highlight the appearance of different instability modes. The calculations are carried out using a three-dimensional CFD of incompressible viscous flow. This study allowed us to observe the birth of a Taylor cell near the upper edge of the flow system. This cell is followed by the generation of spiral waves for different Taylor numbers. The various obtained results are analysed and compared systematically to the experimental results established in the same case.

Keywords: CFD simulation, conical cylinders, instability, laminar-turbulent transition regime, spiral mode

1 Introduction

The flow between coaxial cones has been studied by several authors. Wimmer [1] investigated the appearance of Taylor vortices in different gap configurations, and found that the flow is three dimensional. Noui Mehidi et al. [2] have examined the laminar-turbulent transition in the case of small gap configuration and showed that the flow develops from the laminar regime towards helical motion through the formation of Taylor vortices by varying the angular velocity of the inner cone. Noui-Mehidi et al. [3] showed that different numbers of cells are observed for the limit cases of $\alpha = 0$ and $\alpha = 8^\circ$: the velocity and pressure profiles developing both axially and radially indicate that the outflow boundary cell and the inflow boundary cell do not behave in the same way. Noui-Mehidi et al. [4] studied the effect of wall alignment in a very short rotating annulus, where the outer wall was a cone and the inner wall was a cylinder giving a no-uniform gap. They showed that geometrically broken symmetry can produce flow symmetry for specific combinations of geometrical and dynamical parameters. The motivation of this work, is to investigate numerically the fluid motion in an annulus between cones by using CFD simulations for a three dimensional viscous and incompressible flow.

2 Main problem and modelling

The numerical investigation is conducted with an incompressible viscous fluid contained in the gap between two coaxial cones. The inner cone is rotating with an angular velocity $\Omega$ and the outer one is maintained fixe. Both cones have the same apex angle $\phi = 12^\circ$ giving a constant radial gap so that $\delta = d/R_{1\text{max}} = 0.12$, where $d = R_{2\text{max}}-R_{1\text{max}}$. The fluid is characterized by constant physical properties: density $\rho$, kinematic viscosity $v$. The governing equations characterizing the considered flow are the Navier-Stokes equations.

$$ \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} = -\frac{1}{\rho} \nabla P + v\Delta \vec{V} + f \quad (1) $$

$$ \nabla \cdot \vec{V} = 0 \quad (2) $$
Non-slip boundary conditions are used for radial and axial velocity components. The flow system is completely filled; therefore, the gravity effect is neglected. The control dimensionless parameters used in this investigation are: Taylor number $\text{Ta} = \text{Re} \cdot \delta^{1/2}$ with Reynolds number $\text{Re} = \frac{R_{\text{max}} \cdot \Omega_d}{\nu}$.

By using the finite volumes method the equation system where integrated numerically. A second-order upwind scheme is used to interpolate the face values of the various quantities from the cell centre values for the convection terms in equation (1). The temporal discretization involves integrating all the terms in the differential equations with a time step $\Delta t$ and the integration of the transient terms is implicit by using a second-order formulation. A SIMPLE algorithm is used to link pressure and velocity. When the residual fall below $10^{-6}$ for the pressure and the three velocity components the convergence is obtained.

**Figure 1:** Conical Taylor-Couette flow system

**Figure 2:** Mesh geometry (1512000 cells)

Boundary conditions:

<table>
<thead>
<tr>
<th>Zone</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner cone</td>
<td>Wall ($\Omega \neq 0$)</td>
</tr>
<tr>
<td>Outer cone</td>
<td>wall ($\Omega = 0$)</td>
</tr>
<tr>
<td>Upper end plate</td>
<td>wall ($\Omega = 0$)</td>
</tr>
<tr>
<td>Lower end plate</td>
<td>Wall ($\Omega = 0$)</td>
</tr>
<tr>
<td>Annular gap</td>
<td>Fluid</td>
</tr>
</tbody>
</table>
3 Results and discussions

3.1 Ekman and Taylor vortices

For a Taylor number $0 < Ta \leq 20$, we observe the installation of the Ekman vortex just near the upper part of the flow system; while the second is merged with the larger cell extended along the annular space. The flow regime is always laminar. This observation is consistent with the description of Wimmer (1995) and the experimental results obtained in the case of the annular spaces (Figure 3).

![Figure 3: Streamlines and Ekman cell visualization for $0 < Ta \leq 20$](image)

The Ekman vortex which appears near the upper part of the flow system is characterized by a size $e = 4\text{mm}$, while the rest of the flow forms a large loop with $H-e = 151\text{mm}$. This large loop represents the basic flow of unstable nature (geostrophic instability), which is observed in the flow between coaxial spheres. The radial velocity at the heart of the Ekman cell is maximal and reaches the value $2.10^{-5}$ and vanishes in the vicinity of the inner and outer walls.

![Figure 4: Evolution of the dynamic pressure and the radial velocity versus radial direction](image)

Towards $Ta = 40$ we observe the beginning of formation of a new cell that settles just below the Ekman vortex. This cell is stationary and it formed at $Ta = Tc_1 = 45$ which corresponds to Taylor's vortex. The difference between the numerically obtained Taylor number value is $6.8\%$ compared to the experimental value ($Tc_1=42.3$).

In Figure 5, we observe that the axial velocity in the center of the cell is very high compared to that characterizing the outflow, whereas it is twice the axial velocity corresponds to inflow, we don't find this result in the classic Taylor-Couette (rotating coaxial cylinder).
Figure 5 a): Contour and profile of the axial velocity corresponding to the Taylor vortex

Figure 5 b): Contour and profile of the tangential velocity corresponding to the Taylor vortex

Figure 5 c): Contour and profile of the radial velocity corresponding to the Taylor vortex

Figure 5 d): Contour and profile of the dynamic pressure corresponding to the Taylor vortex
3.2 Downward helical motion

The downward helical motion or spiral mode appears at a critical Taylor number $Ta = T_{c, DHM} = 48$, this value is very close to that obtained experimentally in a small annular gap configuration $T_{c, DHM, expt} = 47$, with a difference of 2.12%.

Figure 5: Streamlines corresponding to the spiral mode

Figure 6: Profile of the dynamic pressure and radial velocity corresponding to the spiral mode

Figure 7: Streamlines corresponding to the spiral mode
By going to Taylor numbers $Ta \geq 60$ we observe the appearance of a secondary cell on the side of the fixed wall at a height $z = 113.5 \text{mm}$. By increasing progressively the Taylor number and at $Ta = 70$, we observe the formation of two secondary vortices one is in the vicinity of the moving wall and the other is next to the fixed wall and that at a height $z = 85 \text{mm}$. These secondary cells will be fused to form a new vortex with a smaller size than the previous ones.

**Conclusions**

After characterizing the different states of flow, it is necessary to compare the results obtained with those established in the experimental investigation.

First, it is noted that the Ekman vortex which appears near the upper part of the flow system is characterized by a size $e = 4 \text{mm}$, while the rest of the flow forms a large loop with $H - e = 151 \text{mm}$. Only the first cell is stationary, it appears at the upper part of the conical flow system. Secondary, cells are generated progressively and inclined relative to the equatorial plane and move in a helical motion (unsteady). There is a formation of two secondary vortices one is in the vicinity of the moving wall and the other is next to the fixed wall. These secondary cells will be fused to form a new vortex with a smaller size than the previous ones. Finally, it is also found that the results are in good agreement with the experiments.

**References**


