ASSESSING QUALITY OF BOUNDARY LAYER FLOWS WITH THE DIAGNOSTIC PLOT

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Abstract

We experimentally investigated turbulent boundary layers over 4 rough surfaces and 1 smooth surface by particle image velocimetry for a range a free stream velocities. The obtained mean velocity vertical profiles were universally scaled by roughness length $z_0$ and friction velocity $u^*$, but both parameters have to be obtained from the fit of the logarithmic or composite velocity profile to the experimental data. Diagnostic plot, which relates turbulence intensity with the mean velocity showed good collapse of different free stream velocities for each surface. It became fully universal in its modified version when the roughness function was taken into an account. We found that turbulence intensities over our surfaces are higher than any published one and that they dependent on roughness length $z_0^+$. 

Keywords: wind tunnel, turbulence intensity, rough-wall, friction velocity.

1 Introduction

The question of how surface roughness affects the turbulent boundary layer (BL) developed above it has not been answered yet. Some studies (e.g., [1], [2]) found the outer-layer similarity for smooth- and rough-wall flows as expected by the Townsend’s hypothesis [3], which suggests that it is only the inner layer (approx. up to five roughness heights) that is affected by surface roughness. On the other hand, there are studies (e.g. [4]) that suggested that the entire boundary layer is affected and that the difference is dependent also on roughness topology. The roughness topology can be very variable, for example: the classical sand paper roughness, a wire mesh, long rods or bars oriented across the flow, or three-dimensional regularly placed obstacles. The highest intensity of turbulence is generated by spatially placed 3D obstacles, which have suppressed along-wind dimension (plates), but roughness surfaces of this kind have not yet been studied systematically.

There are two way how to scale vertical mean velocity profiles. We can use the outer BL scaling based on the boundary layer depth $\delta$ and free stream velocity $U_\delta$ or inner scaling based on friction velocity $u^*$ and viscous length scale $\nu/u^*$, where $\nu$ is kinematic viscosity. Flow over various rough surfaces differ in turbulence intensity, therefore the outer scaling will never lead to an universal scaling. The inner scaling velocity scale $u^*$ suppose to be proportional to the turbulence intensity, so employing it could lead to an universal profile. However, the accurate measurement of friction velocity is difficult (especially on the rough surfaces) and it can have big uncertainties, which can misrepresent measured velocity profiles.

Diagnostic plot was proposed by Alfredsson and Örlü in [5] to judge wall turbulence data near the wall as well as in the outer region. The important feature of the diagnostic plot is that it is not dependent on the wall position and on the friction velocity.

This paper shows systematic study of flow over rough surfaces aiming to find an universal profile or the reason of its failure using different scalings and diagnostic plot.

2 Experimental setup

The measurement campaign was conducted in an open blowing-type wind tunnel of the dimensions 0.25 m, 0.25 m and 4.20 m (width, height and length). Operational wind speeds were in the range 5 to 23 m/s. One smooth and four different rough surfaces were investigated. Setups S4.8, S4.6, S4.4, and S4.2 had the surface roughness made of thin erected plates with dimensions 4 x 4 x 1 mm (width x height x thickness) arranged in staggered rows (Fig. 1). Positions of the roughness
elements are listed in Tab. 1. Setup S0 had no roughness elements but it was manufactured from the same material as the rough ones, therefore it wasn’t perfectly smooth.

<table>
<thead>
<tr>
<th></th>
<th>S4_8</th>
<th>S4_6</th>
<th>S4_4</th>
<th>S4_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance between rows [mm]</td>
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<td>24</td>
<td>16</td>
<td>8</td>
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<td>no. of rows</td>
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<td>168</td>
<td>247</td>
<td>467</td>
</tr>
<tr>
<td>distance between elements [mm]</td>
<td>32</td>
<td>24</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>no. of elements in 1 row</td>
<td>7</td>
<td>9</td>
<td>13</td>
<td>21</td>
</tr>
<tr>
<td>cross-sectional blockage of the roughness elements</td>
<td>0.3%</td>
<td>0.5%</td>
<td>0.7%</td>
<td>1.1%</td>
</tr>
</tbody>
</table>

Table 1: Roughness elements’ positions.

Flow measurements by time-resolved particle image velocimetry (TR-PIV) utilized the diode pumped Nd:YLF laser and Phantom camera (resolution 1280 × 800 pxs). The investigated area covered 140 × 90 mm in vertical and longitudinal direction, it was located on the centreline of the tunnel, and its centre was located 3500 mm from the beginning of the test section, see Fig. 2. The spatial resolution was 0.11 mm per pixel. The data were pre-processed by adaptive correlation algorithm with 16x32 pixels (vertical x horizontal) interrogation area and 50% overlap. The right-handed coordinate system had x-axis oriented with the tunnel axis and z-axis normal to the surface. Each measurement consisted of 4000 double-frame snapshots and sampling frequency $f_S$ and sampling time $T_S$ varied with the free stream velocity to keep the boundary layer turn-over time $T_S U_\infty / \delta \approx 3000$. 

Figure 1: Sketches of investigated surfaces S4_8, S4_6, S4_4, and S4_2. Only parts of the surfaces are shown.
3 Vertical profile of mean velocity

PIV post-processing procedure resulted in 159 x 49 vector matrix in vertical and horizontal direction. The mean vertical profile was calculated as an average of all the vertical profiles in the matrix and it started at the surface and went up with the step of 0.9 mm. Boundary layer depth $\delta_{99}$ was set as the point where the mean velocity is 99% of its free-stream value and $U_\delta$ is the velocity at this level. Friction velocity can be derived from tangential wall stress, but this measurements were not available. Hence indirect estimation of $u^*$ from the vertical velocity profiles or from measured shear stress was used. Flow over a rough wall is also characterized by the roughness length $z_0$ (or alternatively by sand paper equivalent roughness $k_s$ or roughness function $\Delta U$; see [6]).

The value of $u^*_{flux}$ was computed as $\sqrt{|u'u'_w|}$, where $u'u'_w$ is the mean shear stress in the inertial sublayer defined as layer between 0.08 and 0.13 $\delta$ for this purpose.

The values of $u^*_{comp}$ and $z_0$ where obtained by the least-square fitting of the composite vertical profile (adapted from [7]) to the measured vertical profiles. The composite profile is defined as

$$U^+ \equiv \frac{U}{u^*} = \frac{1}{\kappa} \ln \left( \frac{z - d_0}{z_0} \right) + \frac{2\Pi}{\kappa} W(\eta), \quad \eta \equiv \frac{z - d_0}{\delta} \leq 1,$$

where $\kappa = 0.386$ is von Karman constant, $d_0 = 0$ is displacement length, $\Pi$ is known as the wake parameter, and $W$ is the wake function, which is defined to satisfy the normalization conditions $W(0) = 0$ and $W(1) = 1$. We used exponential wake function proposed by Chauhan et al. in [8].

The equivalent expression of Eq.1 using the roughness function $\Delta U^+(z_0) = \kappa^{-1} \ln(z_0^+) + B$ is (see [9])

$$U^+ \equiv \frac{U}{u^*} = \frac{1}{\kappa} \ln \left( \frac{z u^*}{\nu} \right) + B - \Delta U^+(z_0) + \frac{2\Pi}{\kappa} W(\eta).$$

where $B = 5.2$ is the smooth surface additive constant.

The outer and inner scaling parameters for the mean velocity profiles for each surface are listed in Tab. 2. Profiles over the same surface agree very well for different free stream velocities when using the outer scaling, but there are differences for different surfaces (Fig. 3, left). The inner scaling in classical form $U^+ vs. z^+ = z u^*/\nu$ isn’t universal for the rough surfaces since the offset is dependent on $\Delta U^+$ or its equivalent $z_0^+ = z_0 u^*/\nu$ (Fig. 3, right). This dependence is pronounced also in the case of surface S0 which has no roughness elements but due to the natural roughness of the unpolished surface it behaves as aerodynamically rough surface.

Employment of Eq. 1 is more suitable for rough surfaces. The dimensionless velocity $U^+$ is proportional to the fraction $z/z_0$ in the inertial sublayer where the wake part is negligible. There
are two different values of friction velocity listed in Tab. 2. The values are very similar for all profiles, but the importance of their difference is clearly pronounced in Fig. 4. While the surfaces with the roughness elements are insensitive to this difference, the S0 surface scales better with the $u_{\text{comp}}^*$. The variation of the scaled profiles can be caused by uncertainties of the scaling parameters, by uncertainties of the measurement position (relevant close to the wall), and measurement errors.

### 4 Diagnostic plot

The diagnostic plot [5] in the outer region including the inertial sublayer is expressed as

$$\frac{u'}{U} = a + b \frac{U}{U_\delta}$$

where $a$ and $b$ are empirical constants, $u'$ is the standard deviation of the instantaneous stream-wise velocity component, and $u'/U$ is turbulence intensity.
Diagnostic plot for all 35 velocity profiles is plotted in Fig. 5. The profiles show very good agreement for different \( U_\delta \) for each surface, while there are different slopes for different surfaces. S0 setups lay on the line determined from the smooth-wall data compilation in [5] which is described by constants \( a_S = 0.286 \) and \( b_S = -0.255 \) (black dashed line in Fig. 5). Castro et al. in [9] compiled various rough-wall data and found \( a_R = 0.436 \) and \( b_R = -0.389 \) as an upper bound line for all investigated setups (black solid line in Fig. 5). Also Basley et al. in [10] confirmed these values. However, profiles for setups S4.4 and S4.2 have a higher turbulence intensity and the upper bound in our case can be parametrise with \( a_{R2} = 0.49 \) and \( b_{R2} = -0.44 \). Note that the linear dependences are independent of the Reynolds number, only with increasing \( Re \) it is valid in lower elevations (i.e lower \( U/U_\delta \)).

More general scaling of the diagnostic plot for rough surfaces were proposed in [9]

\[
\frac{u'}{U'} = \tilde{a} + \tilde{b} \frac{U''}{U'_\delta}, \quad \text{where} \quad U = U + \Delta U \quad \text{and} \quad U'_\delta = U_\delta + \Delta U. \tag{4}
\]

In the limit for the smooth surfaces \( \Delta U^+ \rightarrow 0 \), Eq. 3 and Eq. 4 will be the same and therefore \( a = \tilde{a} \) and \( b = \tilde{b} \). This modified diagnostic plot is universal for all our vertical velocity profiles as demonstrated in Fig. 5, right.
Figure 5: Turbulence intensity $u'/U$ plotted against the mean velocity. Every third data point is plotted for the chart clarity.

Figure 6: Turbulence intensities at the point where $U/U_\delta = 0.65$, as a function of $k/\delta$, $\delta/z_0$, $z_0^+$, and $Re_\tau$ for all investigated setups. The key in Fig. 5 applies.

The linear part of the diagnostic plot can be characterized by the turbulence intensity at a fixed arbitrarily chosen location, usually in the upper part of the inertial layer, e.g. at the point where $U/U_\delta = 0.65$. This value of turbulence intensity, denoted $u'/U|_{0.65}$ was plotted against various boundary layer parameters in Fig. 6 to reveal which parameter governs the rough-wall behaviour.

Upper left graph in Fig. 6 shows $u'/U|_{0.65}$ as a function of roughness height $k$ over BL depth.
\( \delta \), while the upper right graph in Fig. 6 shows dependence on BL depth \( \delta \) over roughness length \( z_0 \). All parameters \( k, \delta, \) and \( z_0 \) are constant for each surface, so no variability within one surface is visible. The data shows increasing \( u'/U|_{0.65} \) with the roughness density and the smooth data value agree with \( u'/U|_{0.65,s} = 0.12 \) using parameters \( a_{S} = 0.286 \) and \( b_{S} = -0.255 \).

The lower left graph in Fig. 6 shows \( u'/U|_{0.65} \) as a function of \( z_{0}^{+} \) which depends on \( u^{*} \), i.e. on \( U_{\infty} \). There is an increase of \( u'/U|_{0.65} \) with increasing \( z_{0}^{+} \) for all rough setups approaching asymptotic value \( 0.205 \pm 0.008 \) for \( z_{0}^{+} \gtrsim 25 \). Unfortunately such high value of \( z_{0}^{+} \) was obtained only for setup S4_2, and we can’t say if this value is surface-morphology independent.

The lower right graph in Fig. 6 shows \( u'/U|_{0.65} \) as a function of \( Re_{\tau} = u^{*}\delta/\nu \), which can be interpreted as a measure of outer and inner scale separation. Our rough-wall data are very weakly dependent on \( Re_{\tau} \) and it doesn’t show any limit criterion. This is contrary to findings of [6] and [9] where values \( Re_{\tau} = 4000 \) and \( Re_{\tau} = 7000 \), respectively, were suggested as a lower-limit criterion above which the inner scale is at least two order of magnitude smaller than the outer scale. Consequently the inertial sublayer is fully developed and it shields the outer layer from the direct influence of the roughness. However nor of these limit values is evident from our data.

## 5 Conclusion

We experimentally investigated flow over 4 rough surfaces and 1 smooth surface. We didn’t found any Reynolds number dependence within the applied range of the free stream velocities. We found that the mean velocity profiles can be universally scaled by roughness length \( z_0 \) and friction velocity \( u^* \), but both parameters have to be obtained from the fit of the logarithmic or composite velocity profile to the experimental data. Diagnostic plot in its smooth version (Eq. 5) showed good collapse of different free stream velocities for each surface and was fully universal in its modified version (Eq. 6).

The previous studies (namely [9] and [10]) reported maximal asymptotic value \( u'/U|_{0.65,R} = 0.183 \) for cases with \( z_{0}^{+} \gtrsim 15 \) and \( Re_{\tau} \gtrsim 5000 \), however for different type of roughness morphologies (cubes, 2D bars, meshes). Roughness consisted of erected plates is very effective in turbulence generation (strong separation on the edges) and this is probably the reason that maximal asymptotic value reached by our experiment is higher \( u'/U|_{0.65,R2} = 0.205 \). This value was reached after stringent criterion for the roughness length \( z_{0}^{+} \gtrsim 25 \). Such strong demand on high \( z_{0}^{+} \) does relax the demand on \( Re_{\tau} \) since the criterion assures high enough \( u^{*} \) and \( Re_{\tau} \). More experimental data are needed to confirm these findings.

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### References


