GRAVITY EFFECT ON TAYLOR-DEAN FLOW IN CONICAL SYSTEM

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Abstract
The Taylor-Dean flow is a combination of two movements: a Taylor-Couette flow [1] and Dean flow [2]. This type of flow is largely studied theoretically and experimentally [3]. This work dealt with an experimental study about the gravity effect on the onset of the Taylor-Dean instabilities. The experimental setup consists of two horizontal coaxial truncated cones. The inner cone is rotated and the outer one is at rest. The cones have the same apex angle, giving a constant radial gap. According to the variation quasi-static of the angular velocity and the aspect ratio is fixed H/d, one observes by visualization techniques of the flow the effects of the variation of the system inclination angle in the range 0 ≤ α ≤ 90°. Therefore, the gravity effect is evaluated on the appearance of Taylor-Dean instabilities in laminar-turbulent transition regime: primary mode and secondary mode. For a given angular rotation speed, one measures the different characteristics of the movement such that the cell number, the wave number and the phase velocity versus the modified Taylor number.

Keywords: Taylor-Dean flow, gravity effect, primary mode, horizontal coaxial cones.

1 Introduction
Taylor-Dean flow is a combination of: flow between concentric rotating circular cylinders Taylor-Couette flow Taylor [1] and Dean flow with application of a pressure gradient in the azimuthal direction Dean [2]. The Taylor-Dean flow is largely studied theoretically and experimentally due to the simplicity of the geometry and the symmetry properties. The so-called flow has various applications in the industrial sector (manufacture of paper pulp and the galvanizing line in steel mill industry).

The Taylor-Dean instability was considered theoretically by Diprima [3], then Hughes and Reid [4] they showed that the stationary modes do not exist. Thereafter, Raney and Chang [5] showed that the stationary modes are replaced axially by the nonsymmetrical oscillating stationary mode. Gibson and Cuisinier [6] like Seminara and Hall [7] studied the same problem but with small gap configuration. Joo and Shaqf [8] studied the stability of the flow for a viscoelastic fluid (Oldroyd-B) confined between two contra-rotating concentric cylinders with two different speeds Ω₁ and Ω₂. That flow is subjected to an azimuthal pressure gradient. Eagles [10] modified the traditional Taylor-Dean flow system, where the inner circular cylinder is rotating, and the noncircular outer cylinder is fixed. In order to determine the stability characteristics, they have interested to the small gap configuration. In the case of a constant gap or believes in the direction of the basic flow, the flow becomes unstable, conversely, when the gap decreases the flow becomes stable.

Since then, Brewster, Grosberg et Al [11] experimentally treated the problem with presence of the rotation force and azimuth pressure gradient. They were interested in the critical value and the formation conditions of Taylor vortices and they tried to find a common parameter for Dean and Taylor-Couette flows. Other experimental work relating to the Taylor-Dean flow, include those of Mutabazi et Al [12,13] In the case of a primary instability in the model of the inclined rollers in displacement which is simply periodic in space and time. Slightly above this instability, it appear the release of the second instability moved in a periodic modulation of rolls with a high axial frequency and a small wavelength. The rollers inclined in triplet mode, is shown at very low frequency. AitAider [14], made an experimental study on the Taylor-Dean flow between two open transversally coaxial cylinders. Laghouati et Al [15] performed an experimental study of the first modes of instabilities in Taylor-Dean flow in the case of a viscoelastic fluid on the appearance of the two structure modes. They concluded that the wavelength of
the triplet mode corresponded to almost three times the wavelength of the primary mode. Using a visualization technique Daimallah et Al[16] determined the form of vortices as well as the critical parameters corresponding according to various rates of concentration of particles and the aspect ratio of system $\Gamma$.

The aim of this study is to highlight the conditions of appearances of the instability of Taylor-Dean flow in a conical geometry and to compare them with the traditional case (cylindrical geometry). By varying the inclination angle system.

## 2 Experimental device

The experimental device consists of two coaxial cones made of insulating and transparent material (Plexiglas) in order to allow a good visualization of flow regime. Both cones have the same apex angle $\Phi = 12^\circ$ giving a constant annular gap $\delta = d/R_{1\text{max}}$ where $d = (9.68 \pm 0.2)$ mm. The inner cone can rotate and, the outer cone is maintained at rest.

Our system is characterized by an outer cone with largest radius $R_{2\text{max}} = (45 \pm 0.2)$ mm and lowest radius $R_{2\text{min}} = (12 \pm 0.2)$ mm. The largest radius of the inner cone is $R_{1\text{max}} = (35.31 \pm 0.2)$ mm, while the lowest radius is $R_{1\text{min}} = (2.31 \pm 0.2)$ mm. The length of the fluid column is fixed at $H = (155 \pm 0.2)$ mm. A DC motor connected to the rotating axis by a flexible connection in order to avoid the adverse effects of vibration (Figure 1) drives the inner cone.

The working fluid is a solution of 20% of Vaseline oil CHALLALA, favoring a better suspension of the particles in the fluid visualization, which is added to 80% of a petroleum product SIMILI to reduce the viscosity of the oil, with a concentration of 2 g/l of aluminum flakes. Such a mixture constitutes a Newtonian fluid characterized by its kinematic viscosity $\nu = 4.8 \times 10^{-6}$ m²/s and its density $\rho = 777.23$ kg/m³ with an accuracy of 1%.

In order to characterize the onset of the hydrodynamic instabilities, it is necessary to introduce dimensionless numbers involving viscous forces (Table 1), which play a stabilizing role, and centrifugal forces, which have a destabilizing effect. These dimensionless numbers, which serve as control parameters of the flow, are the Reynolds number $Re$, the Taylor number $Ta$ and the Froude number $Fr$ defined in Table 1.

### Reflection of natural light

This method is based on the light beam reflection on the seeding particles. The light is supplied by an external source located in front of the experimental...
device. The intensity of the reflected light will depend on the orientation of the aluminum flakes, which are aligned with the local velocity vector. As a consequence, if the velocity vector is axial, the flakes will reflect light strongly and bright zones will appear on the images. On the contrary, if the velocity has a significant radial component, the flakes will be oriented parallel to the light rays and will let the light pass without reflection, giving dark zones on the images (figure 2).

3 Results and discussions
To proceed, we fixed the height of fluid at $H$ and by varying the flow system inclination angle $\alpha$ by a step of 5°. By increasing the inner cone angular velocity until the onset of Taylor-Dean cells.

The aspect ratio is fixed at $\Gamma = 13.95$ ($H = 135\text{mm}$). By varying the flow system inclination angle $\alpha$ as well as the inner cone speed of rotation $\Omega_1$, we observe:

At $\alpha = 35^\circ$ and $Ta = 286.6$, we observe the appearance of a vortex at the top of the flow system (Figure 3.a) and $\alpha = 40^\circ$, $Ta = 305.6$, we the same phenomena. For $\alpha = 50^\circ$ a twisted cell settled near the border of the free surface (Figure 3.b) at $Ta = 389$. These observations are also confirmed at $\alpha = 55^\circ$.

These cells give rise to twisted Taylor-Dean cells corresponding to an inclination angle $\alpha = 60^\circ$ and a critical Taylor number $Ta = T_{cD} = 351.8$. The twisted cells move in one direction: from the top to the lower part of the flow system. Whereas, for $\alpha = 65^\circ$ and $\alpha = 70^\circ$, the Taylor-Dean cells move one after the other in the same direction.

For $\alpha = 75^\circ$, we observe the appearance of the Taylor-Dean rollers one after the other in the vicinity of the upper part of the device. The Taylor-Dean cells move from left to right, contrary to the previous case. While for a certain value of $Ta = 273.9$ (Figure 4.b), Taylor-Dean rolls tend to move in two opposite directions.

For $\alpha = 80^\circ$ and $Ta = 210.6$, we observe the installation of a vortex near the upper edge of the experimental device (Figure 5.a). By slightly increasing the Taylor number and at $Ta = T_{cD} = 219.4$, we observe the appearance of a finite number ($n = 3$) of Taylor-Dean cells. The rotational speed of the inner cone $\Omega_1$ increases and at $Ta = 323.3$, we observe the occurrence of a source of instability located in the middle part ($h = H/2$) of the device (Figure 4.b). The cells move in two opposite directions from this source with characterized by a wavelength $\lambda^* = 0.524$.

At $\alpha = 85^\circ$, we have observed the appearance of a vortex in the upper part of the device followed by the onset of Taylor-Dean cells at $Ta = 275.2$. When the Taylor number reaches a value $Ta = 286.5$, we have observed two zones regimes: the first zone is affected by the Taylor-Dean cells, which are propagated in both directions from the source of instability. The cells are inclined 12° relative to the vertical position. The second zone is laminar; it is located in the lower part of the experimental device.
Figure 5: Visualization of Taylor-Dean instability at $\alpha = 80^\circ$: a) $Ta = 219.4$, b) $Ta = 323.1$

At $Ta = 426.1$ we observe the superposition of three different zones: a disturbed area followed by a second zone where appears the Taylor-Dean instability and a third one, which is characterized by a laminar regime (Figure 6.a).

Figure 6: Visualization of Taylor-Dean instability at $\alpha = 85^\circ$: a) $Ta = 342.2$, b) $Ta = 432$

At $\alpha = 90^\circ$ and $Ta = 294.2$, we observe the appearance of a baroclinic instability in the upper part of the experimental device followed by the onset of the Taylor-Dean instability and the laminar flow in the lower part.

The cells move from the top to the bottom of the experimental device. For a fixed filling rate $\Gamma = 13.95$ and at $Ta = 218.9$, we observe near the top of the flow system, the appearance of a single cell known as Ekman vortex; the rest of the flow is laminar. At $Ta = 365.8$, it appears seven ($n = 7$) inclined cells where the rollers become dependent on time and the lower part is always laminar. At $Ta = 377$, we observe that some cells move to the left and others separate by moving the source to the right. The Taylor-Dean rollers (figure 7) invade the increasing inner cone rotation speed. For this purpose, we observe the appearance of a disturbance zone in the upper part of the device, another region affected by Taylor-Dean cells while the lower part preserves the laminar regime.

Figure 7: Visualization of perturbed Taylor-Dean flow $\Gamma = 12.90$ and $\alpha = 90^\circ$: a) $Ta = 544.5$, b) $Ta = 748$

In the case of a conical system, we observe the appearance of the primary mode, but the triple mode does not exist. Whereas, this triple mode has already been observed in the case of a cylindrical system (Figure 8).

Figure 8: Visualization of instability of Taylor-Dean between two cylinders a) primary mode $Re = 263$ b) tripled mode $Re = 303$ by Mutabazi [13]
The number of Taylor corresponding to the appearance of the Taylor-Dean instability as a function of the filling rate evolves according to an increasing polynomial law of the 2nd order.

The appearance of the primary mode reveals the existence of a minimum $\alpha = \alpha_c$ included in the interval $75^\circ \leq \alpha_c \leq 80^\circ$ which are shown in the evolution curve of the critical Taylor number as a function of $\alpha$. This result is confirmed by the evolution curve of Froude number $Fr$ versus $\alpha$ which has a maximum located in the same interval $75^\circ \leq \alpha_c \leq 80^\circ$.

For $\alpha = 60^\circ$ the effect of gravity increases to inertia. The value of $\alpha = 77^\circ$ is the critical threshold of preponderance of the gravity effect for $\alpha = 80^\circ$ reduces the inertia effect to the gravity effect. One may question the existence of these particular values of $\alpha$ $60^\circ$ and $80^\circ$. It seems that the Froude number $Fr$ is more sensitive to movement changes. Probably it means that the passage from Taylor-Couette flow to the mixed Taylor-Dean and Taylor-Couette is booted from $\alpha = \alpha_c = 60^\circ$ to $\alpha = \alpha_c = 80^\circ$.

4. Evolution of wavelength for different flow regimes

The evolution of the axial dimensionless wavelength $\lambda^* = \lambda / (2d)$ versus Taylor number for different filling rate $\Gamma$ and different angle $\alpha$ are shown in Figure 10. It is found that the wavelength at different angle $\alpha$ is substantially constant. It can be seen that the wavelength at a different angle $\alpha$ is substantially constant for $\Gamma = 10.84$. In the cases $\alpha = 80^\circ$ and $\alpha = 85^\circ$, we note that the wavelength $\lambda^*$ is almost constant as a function of the Taylor number for a filling ratio $\Gamma = 13.95$.

5 Conclusions

This study has allowed us to highlight the gravity effect on the appearance of Taylor-Dean instability in the flow between rotating coaxial cones. In this case, we observe the appearance of the primary mode, which corresponds to the propagation of the inclined rolls, but the triple mode doesn’t exist as in the
cylindrical system. For a given filling rate $\Gamma$, the critical Taylor number decreases as a function of the inclination angle $\alpha$ between 0° and 80° and it increases for $\alpha$ between 80° and 90° due to the conical shape of the device. For $\alpha = 60°$ the effect of gravity is prevalent compared to inertia. The value of $\alpha = 77°$ is the critical threshold of preponderance of the gravity effect; for $\alpha = 80°$ reduces the inertia effect to the effect of gravity. In particular $\alpha > \alpha_c = 80°$ we note the onset of the primary mode. The wavelength is substantially constant for different filling ratios and different angle. In the future, we will study the evolution of the primary mode by space-time diagram combined with spectral analysis.

References


