NUMERICAL STUDY OF GENERATION OF STEADY STREAMWISE STREAKS IN DUCT FLOW

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Abstract
In this work, the influence of transverse magnetic field on a laminar liquid metal flow in an insulating rectangular duct is numerically solved with three-dimensional (3D) simulations. We use vortex generators as passive mechanism to control the flow. They are small cylinders fixed close to the flow inlet perpendicular to the lower plate that generate vortices at their tip. Simulations are performed by means of the numerical method based on the finite volume. The considered liquid metal is 17Li83Pb eutectic alloy evolving in the rectangular Poiseuille flow. The effect of forced convection and magnetic field is analysed. The velocity variation and shear rate contours at different streamwise positions across the wall-normal direction are presented.

Keywords: magnetohydrodynamics, liquid metal, vortex generators, numerical simulations, velocity.

1 Introduction
The MHD flows in rectangular ducts have been studied since the relevant work of Hartmann [1, 2] where he has carried out the first experimental study of the mercury flow as a conducting fluid in different cross-sections of duct in a homogeneous magnetic field. The development of MHD in engineering has begun since the 1950s. In very interesting papers, Shercliff [3-5] studied experimentally and theoretically the flows in circular pipes with non-conducting walls using the flowmetry. Especially, he developed an approximate method for the laminar motion of a conducting liquid at high Hartmann number for analysing the boundary layers on the walls parallel to the field. He [6, 7] revealed that the axial currents effect promotes the formation of the “M-shape” velocity profile and associated vorticity. Numerous studies have been devoted to the numerical modelling of the MHD flow in different cross-section configurations and the phenomena of magneto-convection were found and discovered which present a diversity of nonlinear effects as pattern formation, instability bifurcations, symmetry breaking, and laminar-turbulent transition [8-12].

It is known that streamwise vorticity and streaks are both main characteristics in the boundary layer. Usually, streamwise vorticity installs as the counter rotating vortex pair in which is developing naturally either through centrifugal instabilities or forcibly by using vortex generators in wall structure. The breaking of fluid flow near the wall region in laminar-turbulent transition regime is one of the most important aspects to understanding the phenomenology of the interaction fluid-structure and shear flow in which significantly affects the momentum, the efficiency of heat and mass transfer.

Several attempts have been made to demonstrate that sinuous modes, which have been detected experimentally and numerically grow by means of the instability mechanism. F. Waleffe [14, 15] analyzed numerically a self-sustaining process (SSP) in wall bounded shear flows for a low and moderate Reynolds number. He studied the linear stability of streaks in a plane Couette flow by using the longitudinal velocity field that restructure the mean shear to generate streaks. He indicated that the instability comes from inflection points in the spanwise direction of streaks evolution. S.C. Reddy et al [16] studied the streak breakdown phase caused by a spanwise inflectional instability of the transition scenarios by a linear stability analysis which occur in plane Poiseuille and Couette flow. They described in detail the streamwise vortex scenario at subcritical Reynolds numbers. Using numerical simulations of nonlinearly saturated optimal streaks, L. Brandt et al [17] focused their work on determination the absolute-convective nature of the secondary instability sustained by crossflow vortices in three-dimensional flat-plate boundary layers. It is revealed that the instability of optimal streaks is clearly convective for all
streak amplitudes and streamwise ranges where the growth of the spatial transient of convective unstable longitudinal velocity streaks leads to the transition of the boundary layer to a turbulent state. The convective nature of the secondary streak instability implies that the type of bypass transition involves streaks that act as amplifiers of external noise. For the transition process, B. Hof et al [18] suggested a model to examine traveling waves in pipe flow. They reported that the dynamics related to pairs of counter-rotating streamwise vortices appear very efficient perturbation that lead to set off the transition scenario to turbulence in a pipe flow. To delay the transition to turbulence using wind-tunnel experiments J. H. M. Fransson et al [19, 20] have treated experimentally the boundary layer in transition to turbulence by introducing small optimal perturbations. They implement appropriate design of roughness elements placed on the skin near the leading edge to create possible vortices. This mechanism is responsible of the amplification and change of the velocity profiles in the bulk flow and to walls vicinity. By means of this passive control technique, it was shown that mechanism of roughness elements can be delaying transition to turbulence. As a result, they proved that these streaks play a stabilizing role on the Tollmien–Schlichting (TS) waves [21]. Through Floquet theory, C. Cossu and L. Brandt [22] analyzed the linear viscous stability of the boundary layer on a flat plate in the presence of nonlinear streamwise streaks which are assumed steady and spanwise periodic. They predicted that the most unstable waves are modified from two-dimensional TS waves into three-dimensional by the presence of the streaks and can have a stabilizing effect on low amplitude streaky-TS waves. They show [23] by numerical simulation that streamwise streaks of large amplitude are able to stabilize TS waves in zero pressure gradient boundary layers at least up to Re=1000. T. Duriez et al [24] characterized experimentally a mechanism of self-sustaining process through streak generation in a flat plate boundary layer. Instantaneous particle image velocimetry PIV field is used for measurement. For moderate Reynolds numbers, they shown experimental evidence of the self-sustaining process between streamwise velocity streaks and streamwise vortices in a flat-plate boundary layer. They showed that a transition occurs for Reynolds numbers between Re$_{c}=720$ and Re$_{c}=900$ which is related with an unsteady modulation of the streaks in the streamwise direction characteristic of the propagation of traveling waves.

In the present study, we examine numerically the combined effect of the aspect ratio and vortex generator solids on the flow structure through cross section of a rectangular duct. We control the transition regime from two-dimensional to three-dimensional flow by varying aspect ratio for different Reynolds numbers. The artificially generation process of the moderate amplitude steady streaks in the boundary layer is more effective for delaying relatively the onset of the viscous instability near-wall. The main objective is to examine the action of combined effect of the roughness devices and magnetic field interaction on the considered flow. Numerical results for different Reynolds number are presented and discussed. The velocity evolution and temperature distributions are provided for different aspect ratio of the configuration of the flow system.

2 Presentation of the problem

We consider the steady flow of an incompressible liquid metal in a rectangular duct with insulating walls and with an externally applied magnetic field perpendicular to flow direction. As depicted in Figure.1, the magnetic is transversal to the plane of the flow and the schematic of the physical and computational domain is shown. The dimensions of the duct are L= 40mm (length), D=5mm (height) E=5-13.5mm (width). VGs are small cylinders defined by a diameter d=0.4mm and height h=0.3mm separating by constant spacing distance λ=1.5mm. The cross section has a rectangular shape; E and D are the rectangular duct characteristics (width-height) perpendicular and parallel to the external magnetic field. The x, y and z are respectively, the streamwise, wall normal and spanwise coordinates. For an incompressible MHD flow in a duct the MHD equation set consists of equations for conservation of mass, conservation of momentum in three directions and the equation governing the electric potential. The magnetic field induces an electric current, which is calculated from the electric potential field. The fundamental MHD equations governing the steady motion of an incompressible electrically conducting liquid-metal fluid in rectangular duct can be expressed as follows;

\[
\rho \left[ \frac{\partial V}{\partial t} + (V \cdot \nabla) V \right] = -\nabla P + \rho \nu \nabla^2 V + J \times B + f
\]

(1)

\[\nabla . V = 0 \quad \text{and} \quad \nabla . J = 0\]

(2)

\[
\frac{J}{\sigma} = - \nabla \phi + V \times B
\]

(3)
\[
\frac{\partial T}{\partial t} + V \nabla T = \frac{K}{\rho c_p} \nabla^2 T
\]

(4)

Where \( V, P, J, B, \varphi \) and \( t \) are velocity, pressure, current density, applied magnetic field, electrical potential and time, respectively. The term \( f \) on the right-hand side of the momentum equation denotes the gravitational force.

Typically, liquid metal MHD flow is characterized by various dimensionless parameters in many applications (fusion reactor). In order to characterize the MHD effect, there are three important dimensionless parameters in MHD flows. The square of Hartmann number \( Ha = BD \frac{B}{\sigma \rho v} \) characterizes the ratio of electromagnetic and viscous forces. The dimensionless parameters defining the stability near the lower wall is Reynolds numbers \( Re_h = V_0 h/\nu \) which is based upon the vortex generator height (h). In this physical conditions \( (R_m << 1) \), it denotes that the induced magnetic field is negligible compared to the applied field. The boundary conditions for the present computation are given as follows: (1) fully developed laminar profile is imposed at inlet. (2) outflow, which is corresponding to the outlet surface of LiPb fluid. (3) No slip boundary condition for velocity at walls.

3 Results and discussions

Figure 2 shows a series of numerical results for different aspect ratio values \( \Gamma = 1, \Gamma = 1.5, \Gamma = 2.1 \) and \( \Gamma = 2.7 \). It presents the velocity evolution of the basic flow for low Reynolds number \( Re_m = 136 \) from a square to the rectangular duct. We examine the duct aspect ratio effect on the flow field in three-dimensional geometry as shown by mean longitudinal velocity lines at four different streamwise locations. The resulting flow structures are symmetric with respect to the middle \((x,y)\) plane. The flow reveals curved streamlines in the first part of the duct as shown at \( x=5\text{mm} \) and \( x=20\text{mm} \) with strong velocity gradients near the walls for all considered cases.

The onset of mushroom-like structure was observed in different configurations at mid-length of the duct \( x=20\text{mm} \). These mushroom structures are obviously developed at the outlet of duct except for square duct \( \Gamma = 1 \) where mushroom hat is not clearly present. The coupling of gravitational force, thermal gradient between the hot fluid and the cooling wall and pressure force acting according to the imposed velocity value causes their installation. For this small Reynolds number, the vortex generators have an influence just at the inlet of the duct observed through the formation of spanwise undulation as shown by the isolines of longitudinal velocity for \( x=5\text{mm} \). For each geometry, we found that the number of created spanwise modulation at the upstream duct is the same vortex generators number. It is noticed that when the aspect ratio is increased the flow and mushroom-like structures are expanded within \( z \)-direction. The hat and stem of mushroom are larger and flattened when aspect ratio or spanwise direction is increased.

We remark that the case with aspect ratio \( \Gamma = 1.5 \) displays a good form of mushroom as reported and observed in previous literature.
Figure 2: Flow patterns for low Reynolds numbers $Re_{hc}=136$ for different aspect ratio values $\Gamma = 1$ to $\Gamma = 2.7$.

Figure 3 shows the mean flow and velocity contours for medium and large aspect ratios $\Gamma = 2.1$ and $\Gamma = 2.7$ for different Reynolds numbers in perpendicular planes to the VGs devices for $y=h=0.3\text{mm}$ corresponding to the VGs height and $y=h/2=0.15\text{mm}$ corresponding to the VGs mid-height. It is very important to control the flow close the lower wall where VGs elements are positioned near the leading edge. By comparing the aspect ratio impact on the flow field for moderate and large Reynolds numbers we found that VGs have no effect for Reynolds number less than $Re_{h}=300$ and the critical Reynolds number showing the fully growth of counter rotating streamwise vortices from the inlet to the outlet of the rectangular duct is detected at $Re_{hc1}=543$, $Re_{hc2}=1087$ and $Re_{hc3}=1358$ for $\Gamma = 1.5$, $\Gamma = 2.1$ and $\Gamma = 2.7$ respectively. The onset of steady streaks is delayed when the aspect ratio is increased. The velocity evolution in spanwise direction shows a symmetric distribution with respect to the middle $(x,y)$ plan in all cases. The VGs effect is more pronounced at VGs level than at middle VGs level due to the imposed no-slip condition at the lower duct wall.

Figure 3: Velocity contours for different Reynolds numbers in different $(x,z)$ plane at $y=0.15\text{mm}$ and $y=0.3\text{mm}$ with VGs: (a) $\Gamma = 2.1$ and (b) $\Gamma = 2.7$.
Figure 4 shows the $\partial u/\partial y$ and $\partial u/\partial z$ shear rate contours at different streamwise cross-section positions across the wall-normal direction for Reynolds numbers $Re_n=1087$. The strong shear regions at the lower plate wall are observed. High positive shear region corresponds to the inflection point in the velocity profile close to the boundary layer. Strong inflectional velocity profiles in the spanwise and normal directions is detected which are associated to the strong regions of oscillation. The results obtained for $\partial u/\partial z$ are shown in Figure 4c. It displays that the regions of high $\partial u/\partial z$ shear that are created near the lower plate. It is observed that each structure exhibits two counter rotating symmetric vortices where the iso-shear contours displaying the strong positive and negative of shear rate concentrations from the inlet to the outlet of the duct. We indicate that for x-location varies in the range $5\text{mm}\leq x\leq 15\text{mm}$ small eight vortices are generated and beyond the $x=20\text{mm}$ they are larger than the first ones.

Figure 4: Shear rate contours on $(y,z)$ plane for different streamwise positions for $Re_n=1087$ and $\Gamma=1.5$: (a) iso-shear contours of $\partial u/\partial y$ and (b) iso-shear contours of $\partial u/\partial z$.

Figure 5a provides the velocities as function of the spanwise component $z$ at a distance $y=2.5\text{mm}$ (solid line) and $y=3.5\text{mm}$ (dashed line) from the lower plate duct wall for low Reynolds number $Re_n=136$. Two maxima of velocity are shown near the sidewalls. On the other part, we can notice that the hat of mushroom is presented by a sink between two velocity maxima where $z=3.75\text{mm}$ determines the position of the center of the two counter-rotating cells. The spanwise solid line confirms the presence of one wave in $z$-direction and it is important at width duct half $y=2.5\text{mm}$ than close to the wall at $y=3.5\text{mm}$. The velocity vectors illustrate the movement of fluid particle from the bottom wall to the head of the mushroom. The velocity is concentrated near the mushroom stem as shown by vectors in difference direction generating two sources vortices, fig.5b.

Figure 5: Velocity profile for $Re_n=136$: (a) velocity profile versus the spanwise component at the a distance $y=2.5\text{mm}$ and $y=3.5\text{mm}$ (b) velocity vectors at the exit of the duct ($x=40\text{mm}$).
Figure 6 shows the VGs influence on spanwise velocity field at y=0.15mm for Reynolds numbers Re=1087 and Re=1358 for \( \Gamma = 2.1 \) and \( \Gamma = 2.7 \) respectively. This velocity evolution indicates the critical formation of streamwise structures (streaks) that are created at downstream of the duct x=40mm. It depicts a speed variation in the spanwise direction that is associated with fluctuation of the streaks as observed in vortical structures and the flow is characterized by spanwise maxima that are propagated as oscillating waves.

It can be seen that the periodic velocity modulations is symmetric to the (x, y) plane and the VGs effect decreases with the increase of duct length position. The differences in velocity oscillations provide a strong perturbation of boundary layer in streamwise direction.

Figure 7 shows the velocity distribution in different planes without and with the magnetic field for Reynolds number \( R_{hc2} = 1335 \) and \( \Gamma = 1.5 \). In the hydrodynamic case (B=0), the mean flow was maximum in the core flow due to the inlet boundary condition and it was unstable with longitudinal velocity streaks near the walls due to the VGs devices where the nonlinearities deform the spanwise velocity profile. When a transverse magnetic field was imposed, the velocity distribution in the duct changes strongly. It is decreased in the bulk flow for (y, z) planes.

Furthermore, the velocity profiles in the (x, z) plane at y=0.15mm under the influence of the transverse magnetic field are presented in Figure 8. When the magnetic field is increased, the spanwise velocity increases strongly as compared to the reference case (B=0). We note that the magnetic field has a sensitive impact on the velocity profile near the lower plate perpendicular to the magnetic field when B is increased. The Lorentz force acts on the flow in the streamwise direction, which gives rise to thin velocity boundary layers with high velocity gradients near the walls.

Figure 7: Velocity distribution without and with a magnetic field for \( R_{hc2} = 1358 \), \( \Gamma = 1.5 \)
4 Conclusions

In the present work, the focus of the study is on the combined effect of the aspect ratio and magnetic field on the flow structures produced by roughness vortex generators elements. The formation mechanism of counter rotating streamwise vortices created by forced VGs devices that are mounted near the leading edge is explored. For different aspect ratios $1\leq \Gamma \leq 2.7$, the onset of mushroom like structure generated by buoyancy in the bulk flow is found at the same critical Reynolds number $Re_h=136$. When the aspect ratio is increased, the head and stem of mushroom are larger and become flattened. It is found that the mushroom structure dominates the flow at low Reynolds number $Re_h \leq 272$ where the VGs effect is not yet prominent. The critical Reynolds number displaying the totally development of counter rotating streamwise vortices in longitudinal direction is noticed at $Re_h=543$, $Re_h=1087$ and $Re_h=1358$ for $\Gamma=1.5$, $\Gamma=2.1$ and $\Gamma=2.7$ respectively. When the aspect ratio is increased, the installation of steady streaks is delayed. For a large Reynolds number, the flow regime is not stable downstream of the duct and its development leads to the appearance of horseshoe vortices. The VGs affect strongly the boundary layer by a spanwise modulation. The $\partial u / \partial y$ and $\partial u / \partial z$ shear rate contours at different streamwise cross-section positions across the wall-normal direction for Reynolds numbers $Re_h=1087$ are presented. The velocity distribution in the duct changes strongly when a transverse magnetic field is applied and breaks then the nonlinearities in the spanwise velocity profile.

References


