Experimental investigation of the inlet boundary layers of a linear blade cascade is topic of this contribution. Experiments, that are present here, were performed as part of the research of the flow in the linear blade cascade and results will be used for the future evaluation of the end-wall losses in the cascade. Two configurations of the cascade were tested, which were defined by pitch/chord ratio (t/c). Based on outlet isentropic Reynolds number $Re_{2is}$, several regimes of flow were investigated for each configuration. Measurements were performed by means of Pitot tube. Experimental results were compared with the theory of the boundary layer.

Keywords: Boundary layer, linear blade cascade, pressure measurement

1 Introduction

The kinetic energy losses in the turbomachines are essential topic because of its straight connection to the turbine efficiency. A Comprehensive review about the losses generating mechanism in turbomachines was published by Denton [1]. Denton divided losses in the turbomachine into three parts:

- Profile losses - these losses are generated by the boundary layers on the blade surfaces and include also extra losses generated at the trailing edges of the blades.

- Endwall losses - are generated as the result of the interaction between the leading edges of the blades and boundary layers on the endwall of the channel and due to the passage vortex and corner vortices. Because of this interaction, vortex structures are generated before the leading edges of the blades as consequences of the momentum balance. More information about the endwall losses can be found in Lampart [2, 3].

- Tip leakage losses - these losses are generated by the leakage of fluid over the tips of the rotor blades and the hub clearance of the stator blades. Mechanism of the loss generation in this case is dependent on whether the blades are shrouded or unshrouded.

The profile losses are usually determined from the flow parameters measurement behind the linear blade cascade. The measurement is provided by the multi-hole pressure probe traversing through several blade-to-blade channels in the central plane in some distance behind the trailing edges of the blades.

For the end-wall losses evaluation, whole flow field behind the cascade must be measured in several planes along the blades height. For the complete evaluation of the end-wall losses, character of the inlet boundary layer on the end walls of the test section in front of the blade cascade must be also known. The aim of the presented work is to evaluate the inlet end-wall boundary layers in front of the linear blade cascade at different Reynolds numbers.

2 Boundary layer investigation

Prandtl in 1904 [4] divided flow field into two parts: the boundary layer (BL), that is the thin layer close to the body surface, where the viscous force play an important role and rest of the flow field, where the viscous force can be neglected. Many information about BL can be found in Schlichting & Gersten monograph [5].
2.1 Boundary layer thickness and shape parameter

Boundary layer thickness is important parameter and can be defined by many ways. Experimental thickness of the BL $\delta$ is defined as $y = \delta$ if $u = 0.99U_\infty$, where $y$ is the normal coordinate to the body surface, $u$ is the velocity in the BL and $U_\infty$ is the free stream velocity. For laminar and turbulent BL on the flat plate, the experimental thickness of the BL can be approximately calculated from the following relations:

$$\delta = 5.84x/\sqrt{Re_x} \quad \text{laminar BL},$$

$$\delta = 0.37x/5\sqrt{Re_x} \quad \text{turbulent BL},$$

where $x$ is the distance from the leading edge of the plate and $Re_x$ is the Reynolds number defined as $Re_x = ux/\nu$, where $\nu$ is the kinematic viscosity. The transition between two BL regimes (laminar/turbulent) on a flat plate occurs in the range of $Re_x \in (0.5 - 2) \times 10^6$. This value is dependent prevailing on the surface quality, for example on roughness.

Another definition is displacement thickness. For the compressible fluid, it is defined as:

$$\delta^* = \int_0^\infty \left(1 - \frac{\rho u}{\rho_e U_\infty}\right) dy,$$

where $\rho$ is the fluid density in the BL and $\rho_e$ is the fluid density of the free stream.

Next definition of the BL thickness is momentum thickness defined as:

$$\delta^{**} = \int_0^\infty \frac{\rho u}{\rho_e U_\infty} \left(1 - \frac{u}{U_\infty}\right) dy.$$

Based on these two definitions of the BL thicknesses, shape parameter can be defined as ratio of those two in a form:

$$H_{1.2} = \frac{\delta^*}{\delta^{**}}.$$

According to value of this parameter, type of the BL can be estimated. For the laminar BL, value of $H_{1.2} \approx 2.59$ is valid and for turbulent BL, value of $H_{1.2} \approx 1.34$ is valid. All of the definitions mentioned above can be found in Schlichting & Gersten [5].

2.2 Law of the wall

Based on wall shear stress $\tau_w$ and fluid density, friction velocity can be introduced as $u_\tau = \sqrt{\tau_w/\rho}$.

Using Prandtl’s mixing length for turbulent BL logarithmic law of the wall can be derived in form:

$$u/u_\tau = 1 - K \ln \left(\frac{yu_\tau}{\nu}\right) + K,$$

where $\chi$ is the Kármán constant and $K$ is constant. Kármán constant was theoretically established by Baumert [6] as $\chi = 0.399$ and constant $K$ is dependent prevalent on roughness of the wall, see Schlichting & Gerten [5]. There are some doubts about the value of Kármán constant and its invariance, see George [7], Bailey et al. [8] and Örlü et al. [9] summarized different values of constants $\chi$ and $K$ from the literature. Values of these constants can vary according to these two papers under different conditions, $\chi$ in the range $0.33 < \chi < 0.43$ and $K$ in the range $3.5 < K < 6.1$.

Based on equation (4), dimensionless velocity can be defined as $u^+ = u/u_\tau$ and characteristic wall coordinate as $y^+ = yu_\tau/\nu$, see Schlichting & Gerten [5]. Also ranges in which the law of the wall is valid are discussed in literature, see Marusic et al. [10].

In present paper dynamic viscosity of the air was evaluated using Sutherland’s formula [11]:

$$\eta = \eta_0 \left(\frac{T}{T_0}\right)^{\frac{3}{2}} \frac{T_0 + T_s}{T + T_s},$$

where $\eta_0 = 1.716 \cdot 10^{-5}$Pa/s, $T$ is the thermodynamic temperature, $T_0 = 273.15$ K is the reference temperature and $T_s = 110.4$ K is the Sutherland’s temperature. Kinematic viscosity was then calculated as $\nu = \rho \eta$. 
2.3 Wall shear stress evaluation

Wall shear stress can be evaluated from the Pitot tube measurement. This method was proposed by Preston in [12]. Many authors studied applicability and limitation of this method, see Patel [13], Sutardi and Ching [14] and Bechert [15]. Based on calibration of the Preston tube, relation between wall shear stress $\tau_w$ and measured pressure difference $\Delta p$ can be established. Some calibration equations were published by Sutardi and Ching [14]. Equation, that was used for wall shear stress evaluation, was adopted from Bechert [15], because of its universality, see Sutardi and Ching [14]. Bechert’s equation has a form:

$$\tau^+ = \frac{4}{\sqrt{28.44}} (\Delta p^+) ^2 + 6.61 \times 10^{-6} (\Delta p^+) ^{3.5},$$

where $\tau^+ = \tau_w d^2/(\rho \nu^2)$ and $\Delta p^+ = \Delta p d^2/(\rho \nu^2)$, here $d$ is the Preston tube diameter.

After the wall shear stress is determined, the skin friction coefficient can be calculated as:

$$c_f = \frac{2 \tau_w}{\rho U^2}.$$  \hspace{1cm}(7)

Skin friction coefficient can be also calculated from the relationship:

$$c_f = 2 \left[ \frac{\chi}{\ln Re_x} G \left( \ln Re_x \right) \right]^2,$$

where according to Schlichting & Gersten [5] $G \approx 1.5$ in the range of $10^5 < Re_x < 10^6$.

2.4 Preston tube position

If the Preston tube is placed near the wall, streamlines are shifted due to its presence (blockage effect). If the Preston tube is placed to the velocity gradient, pressure that is indicated by the probe is higher than true pressure, that’s because of the averaging of the pressure across the probe face. From that reasons some kind of corrections for Preston tube position must be applied. Velocity gradient correction was proposed by MacMillan [16] in the following form:

$$\Delta y = \epsilon d,$$

MacMillan [16] suggested that $\epsilon$ should be constant, but that is hardly true, so McKeon at al. [17] proposed for $\epsilon$ relation in form:

$$\epsilon = 0.15 \tanh (4 \sqrt{\alpha}),$$

where $\alpha$ is defined as:

$$\alpha = \frac{d}{2 u} \frac{du}{dy}.$$  \hspace{1cm}(11)

Near wall correction was adopted from Bailey at al. [18] in form:

$$\epsilon_w = A \left( \frac{y}{d} - 3 \right) + B \left( \frac{y}{d} - 3 \right) \left[ 0,15 \tanh (4 \sqrt{\alpha}) \right],$$

where $A = 0.174$ and $B = -1.25$ are constants. This correction should be used in the range of $0 \leq y \leq 3d$.

Then correction for the Preston tube position is composed from the two corrections mentioned above:

$$\Delta y_{tot} = \epsilon_{tot} d = (\epsilon - \epsilon_w) d.$$  \hspace{1cm}(13)

Another correction for Preston tube position can be found in literature, see Zagarola and Smith [19] and Bailey at al. [18]. These corrections (viscous and turbulence corrections) were not used in this paper because of experimental setup.
### 3 Experimental setup and apparatus

Experiments were performed in the VZLU laboratory of high speed aerodynamics in a closed loop low-pressure WT. Scheme of the WT test section is shown in Figure 1. In the figure, measured pressure differences are also shown. Scheme of the whole wind tunnel is shown in Figure 2.

Two configurations of the blade cascade were investigated. The configurations were marked by a pitch/chord ratio $t/c = 0.6$ and 0.9, respectively. The cascade was assembled of the individual prismatic blades placed between the two test section windows made of plexiglass, that is unique for each blade configuration. The number of blades in the cascade depends on the pitch/chord ratio. The flow incidence can be adjusted, with respect to the fixed cascade, by a pair of shaped semi-nozzles upstream of the linear cascade. Inlet flow angle was set at constant value $\alpha = 85^\circ$. Outlet isentropic Mach number $M_{2ix}$ was also constant for all tested regimes, and its value was $M_{2ix} = 0.4$. Five different Reynolds numbers of flow were investigated for configuration $t/c = 0.6$ and four for configuration $t/c = 0.9$. For each regime, three positions in the $z$ direction were measured. Positions were chosen with respect to the blade position in the cascade. The first and the third position were measured in front of the leading edge of the blade and second was measured in the middle of the channel. BLs were measured in the middle of the cascade (i.e. in the $7^{th}$ channel for the configuration $t/c = 0.6$. and in the $5^{th}$ channel for the configuration $t/c = 0.9$). Regimes were set according to the outlet isentropic Reynolds number, that was defined as $Re_{2ix} = u_\infty c/\nu$. For detailed experimental settings see Table 1. Tested regimes are marked as: $Rwx_y-z$, where $w$ is mark for a flow regime in the WT, $x = a$ or $b$ are the marks of configuration $a$ for $t/c = 0.6$ and $b$ for $t/c = 0.9$, $y = 1$ or 2 are the marks for probe pressure tap and $z$ represents the probe position in $z$ direction, see Figure 1. The change of $Re_{2ix}$ (flow regime) can be achieved by reduction of the pressure in the WT by the exhaustion of the air using a vacuum pumps.

For the measurement in the BL, Preston tube of 0.6 mm outer diameter and with inner diameter of 0.4 mm was used.

Barometric pressure was measured by means of Druck DPI 145 with accuracy of 0.013% FS. Pressures in the BL were measured by means of Preston tube, see Figure 1. Reference total pressure at the inlet of test section was measured in the relaxation chamber and static pressure at the wall in front of the cascade. All pressures were converted to voltage output using pressure transducers with accuracy of 0.1% rdg. The measured signals were sampled using A/D card (National Instruments PCI-6259 A/D card: 16-Bit, 1MS/s (Multichannel), 1.25 MS/s (1-Channel), 32 Analogue Inputs). The rate could be set according to frequencies of the observed process. Signal conditioning was provided by DEWETRON system, which was equipped with modules with bandwidth up to 200 kHz.

According to the measurement device accuracy, uncertainty of the measured quantities were established according to the *GUM* document [20]. For the outlet isentropic Mach number $M_{2ix}$, the uncertainty was less than 1% as well as for inlet isentropic Mach number $M_{1ix}$. The uncertainty of the free stream velocity $U_\infty$ did not exceed 1.1% and for the velocity in the BLs less than 3.3%.
Table 1: Flow regime settings and theoretical values of $\delta$.

<table>
<thead>
<tr>
<th>Regime</th>
<th>$R_{2is}$</th>
<th>$R_{x}$</th>
<th>$\delta$ [mm]</th>
<th>Regime</th>
<th>$R_{2is}$</th>
<th>$R_{x}$</th>
<th>$\delta$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1a</td>
<td>$4.40 \times 10^5$</td>
<td>$1.32 \times 10^6$</td>
<td>16.4</td>
<td>R1b</td>
<td>$4.45 \times 10^5$</td>
<td>$1.47 \times 10^6$</td>
<td>16.2</td>
</tr>
<tr>
<td>R2a</td>
<td>$2.50 \times 10^5$</td>
<td>$0.74 \times 10^6$</td>
<td>18.5</td>
<td>R2b</td>
<td>$2.50 \times 10^5$</td>
<td>$0.83 \times 10^6$</td>
<td>18.2</td>
</tr>
<tr>
<td>R3a</td>
<td>$1.20 \times 10^5$</td>
<td>$0.35 \times 10^6$</td>
<td>7.4</td>
<td>R3b</td>
<td>$1.22 \times 10^5$</td>
<td>$0.41 \times 10^6$</td>
<td>6.9</td>
</tr>
<tr>
<td>R4a</td>
<td>$0.80 \times 10^5$</td>
<td>$0.26 \times 10^6$</td>
<td>8.4</td>
<td>R4b</td>
<td>$0.77 \times 10^5$</td>
<td>$0.21 \times 10^6$</td>
<td>8.2</td>
</tr>
<tr>
<td>R5a</td>
<td>$0.70 \times 10^5$</td>
<td>$0.21 \times 10^6$</td>
<td>9.4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Based on 95% confidence level ($\pm 2$ standard deviation).

4 Results and discussion

For both configurations, measurements were performed 29 mm in front of the leading edges of the blades. Measured points were chosen with respect to the probe dimension and the size of the traverse mechanism step (132.5 step per 1 mm). In the near wall region (BL), distance between the measured points was 0.2 mm. With increasing distance from the wall, the step of the measurement was increasing too with respect to the theoretical calculation of the BL thickness. These values were calculated using relationship that were introduced in section 2.1. Theoretical thicknesses of BLs are summarized in Table 1.

4.1 Results for configuration $t/c = 0.6$

In Figure 3 dependencies between $u^+$ and $y^+$ are shown in semilogarithmic coordinates for regimes R1a and R2a. In this figure, measured data are approximated by the logarithmic law of the wall (Eq. (4)), where values of the constants were established as a best fit of measured data and the values were determined as $\chi = 0.4$ and $K = 4.3$. As can be seen from Figure 3, theoretical approximation is in good agreement with measured data, so the BL in this case can be characterized as turbulent. For these two regimes thicknesses of the viscous sublayers were also calculated using:

$$\delta_v = \frac{50}{R_{x} \sqrt{\chi}}$$  (14)

Thicknesses of the viscous sublayers were determined according to Eq. (14) as $\delta_v = 0.06$ mm for regime R1a and $\delta_v = 0.1$ mm for regime R2a. Width of the probe wall was 0.1 mm (see section 3), thus this probe can not be used for the measurement in the viscous sublayer. According to Schlichting & Gersten [5], viscous sublayer is within the $0 < y^+ < 5$, which is out of the region of our measurements. In Figure 4 dependencies, between $u^+$ and $y^+$ are shown in semilogarithmic coordinates for regimes R3a, R4a and R5a. Data in these cases were approximated using relation $u^+ = y^+$, which is the relationship for laminar BL. As can be seen from the figure this approximation is in good agreement with measured data, hence BLs on the wall can be pronounced as laminar.

Shape parameters $H_{1,2}$ were within the 1.2 and 1.4 for regimes R1a, R2a, hence BLs on the wall can be marked as turbulent event from this evaluation. For regimes R3a, R4a and R5a shape parameters $H_{1,2}$ were within the 2 and 2.5, so the BLs were marked as laminar as well as from previous evaluation.

4.2 Results for configuration $t/c = 0.9$

Dependencies between $u^+$ and $y^+$ for the regimes R1b and R2b are shown in Figure 5. Measured data were approximated using law of the wall, in this case values of the constants were established as a best fit of measured data and the values were determined as $\chi = 0.36$ and $K = 4$. Discrepancy of constants for tested configurations may have been caused by re-adjustment of the probe after...
Figure 3: Comparison of the velocity profile for regimes R1a and R2a with logarithmic law of the wall.

Figure 4: Comparison of the velocity profile for regimes R3a, R4a and R5a with relation $u^+ = y^+$.

Figure 5: Comparison of the velocity profile for regimes R1b and R2b with logarithmic law of the wall.

Figure 6: Comparison of the velocity profile for regimes R3b and R4b with relation $u^+ = y^+$. 
the exchange of the blade cascade. Slight change of the position of the probe can significantly effect the evaluation.

As seen from Figure 5 for the regimes R1b and R2b, BLs were also turbulent, compared to the regimes R3b and R4b, where the BLs were laminar, see Figure 6.

Shape parameters $H_{1,2}$ were within the 1.3 and 1.6 for regimes R1b and R2b, hence BLs on the wall can be marked as turbulent. For regimes R3b and R4b shape parameters $H_{1,2}$ were within the 2.3 and 2.5, so the BLs were marked as laminar.

4.3 Skin friction coefficients and correction of the probe position

For both configurations, skin friction coefficients were determined for each flow regime, see Figure 7. In the figure, evaluated data are compared with theoretical calculation (Eq. (8)). Calculation was performed with Kármán constant $\chi = 0.399$, that is value determined by Baumert. As can be seen from the figure, skin friction coefficients determined from the measurements copy the trend of the theoretical evaluation. It can be seen from the figure that for regimes R1a, R1b, R2a and R2b all evaluated coefficients had approximately same values for all measured positions $z$. Other regimes did not show such good match, because of the uncertainties of measurements (measured pressure differences $\Delta p$ had lower values, so the uncertainty of the coefficients increase). In Figure 8,

![Figure 7: Friction coefficients](image)

![Figure 8: Probe position before and after Bai-leys correction.](image)

influence of the probe position correction is shown for regime R1a (for all other regimes the results were comparable with this example). As can be seen from the figure, with increasing distance from the wall the corrected positions draw closer to the positions indicated by the traverse mechanism.

5 Conclusion

The boundary layers on the walls of the WT test section at the inlet of the linear blade cascade were measured. Two configurations of the cascade were tested, $t/c = 0.6$ and $t/c = 0.9$, respectively. From the results, type of BLs on the walls were determined based on the evaluated dependencies between the $u^+$ and $y^+$ and from the shape parameters $H_{1,2}$. Turbulent BLs were observed during the flow regimes R1a, R2a, R1b and R2b, compared to regimes R3a, R4a, R5a, R3b and R4b, when laminar BLs were found. Results are in agreement with theory of BL on a flat plate, where the Reynolds numbers $Re_x$ and shape parameters $H_{1,2}$ determined from measurements agree with theoretical values.

It can be seen from the results, that Preston tube was not able to measure in the viscous sublayers, due to its dimension. For the measurement in this region, some other method should be used (thermal anemometry), if the evaluation of the viscous sublayers was needed.

Results from this experiments will be used for the evaluation of the endwall losses of the cascade in the future research.

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