UNSTEADY VORTEX DYNAMICS PAST A UNIFORMLY MOVING TILTED PLATE

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Abstract

In the present paper, stratified fluid flows past a uniformly moving tilted plate are studied numerically at transient vortex regime based on the fundamental system of differential equations for incompressible stratified fluid. Numerical simulation of the problem is implemented using dynamic mesh libraries and unique solvers of own development in frame of the OpenFOAM package with open source. Unsteady patterns of flow vortex structure past a moving plate are thoroughly studied and analysed at different tilt angles of the plate with a purpose to grasp the general multiscale process of flow evolution over time at the initial stage of the flow formation. Based on observations of the unsteady flow patterns and evaluations of spatial and temporal scales of the vortex elements, as well as their geometrical features, manifestation level, and dissipation rate, a number of vortex flow regimes are distinguished depending on a value of tilt angle of the plate to horizon. A particular attention is paid to studying the flow fine structure which is a key for explaining plenty of natural phenomena still not clearly understood.

Keywords: stratified flow, tilted plate, vortices, vortex sheet, fine structure, OpenFOAM

1 Introduction

The flow field around bluff bodies, such as plates, discs, circular and rectangular cylinders and V-shaped prisms, has been extensively studied, due to its relevance to drag on vehicles, ship hulls and submarines. Such flows provide rich and interesting fluid dynamics of considerable engineering relevance, including usage in combustors to enhance scalar mixing and to provide a flame-stabilizing region [1] and in air diverters, enabling hovercraft fans to determine both vertical and horizontal thrusts [2, 3]. A number of investigations on some classical configurations have been done, both experimentally and numerically, in order to understand the fundamental aspects of flow structure around bodies and in the wake behind them. One of the earliest works about vortex shedding from a sharp-edged plate was performed by Fage and Johansen [4], who analyzed the flow field around a flat plate for 18 different angles of incidence. Jackson [5] simulated the periodic behavior of a two-dimensional laminar flow past various shaped bodies, including flat plates aligned over a range of angles of attack with respect to the incoming free-stream. Lam [6] investigated the flow past an inclined flat plate at θ=15°, using phased-averaged LDA measurements. The flow field around flat plates, characterized by sharp leading and trailing edges, was also investigated by Breuer and Jovicic [7]. Breuer et al. [8] simulated the flow over an 18° inclined plate, showing how the trailing edge vortices are able to dominate the wake features. Zhang et al. [9] studied the transition route from steady to chaotic state for a flow around an inclined flat plate.

It should be also mentioned the flow structure and dynamics around an obstacle is essentially dependent on real properties of a medium, as well. In the environment, i.e. the Earth’s hydrosphere and atmosphere, and different industrial devices, fluid density, as a rule, is not constant due to inhomogeneity of either soluble substances or suspended particles concentration or temperature and pressure distributions [10]. Under the action of buoyancy forces fluid particles with different density move vertically and form a stable stratification with buoyancy period which can vary from a several seconds in laboratory conditions and up to ten minutes in the Earth’s atmosphere and hydrosphere [11].

The present investigation focuses on the numerical analysis of the unsteady multiscale stratified flow around a uniformly moving tilted plate in a wide range of tilt angles to horizon based on numerical solution of the fundamental system of fluid mechanics equations, which allows studying flows of both continuously stratified and homogeneous viscous incompressible fluids in a single formulation.
Movement of a plate is implemented using dynamic mesh techniques in frame of the OpenFOAM package that made it possible to reproduce exactly the conditions of the laboratory experiments in the LFM IPMech RAS [12], avoid arising non-physical perturbations at the external boundaries of the computation domain and describe adequately the flow structure and dynamics at the initial stage of the plate movement. This improves the results previously obtained by the authors on unsteady stratified flows around horizontal and sloping strips in the linear [13] and complete nonlinear formulations [14].

2 Problem formulation and solution

Mathematical modeling of the problem on flows around a sloping plate is based on the fundamental system of equation for multicomponent inhomogeneous incompressible fluid in the Boussinesq approximation. The buoyancy and diffusion effects of stratified components are taken into account, while the effects of heat-conductivity and heating due to dissipation are neglected [15]. Thus, the governing equations take the following form,

\[
\begin{align*}
\rho &= \rho_0 \left( \exp \frac{-z}{\Lambda} + s \right), \\
\nabla \cdot \mathbf{v} &= 0, \\
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \nabla) \mathbf{v} &= -\frac{1}{\rho_0} \nabla P + \mathbf{v} \Delta \mathbf{v} - s \cdot \mathbf{g}, \\
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla s &= \kappa_s \Delta s + \frac{v_s}{\Lambda}.
\end{align*}
\]

Here, \( s \) is the salinity perturbation including the salt compression ratio, \( \mathbf{v} = (v_x, v_y, v_z) \) is the vector of the induced velocity, \( P \) is the pressure except for the hydrostatic one, \( \mathbf{v} = 0.01 \text{cm}^2/\text{s} \) and \( \kappa_s = 1.41 \times 10^{-5} \text{ cm}^2/\text{s} \) are the kinematic viscosity and salt diffusion coefficients, \( t \) is time, \( \nabla \) and \( \Delta \) are the Hamilton and Laplace operators respectively.

Physically valid initial and boundary conditions are no-slip and no-flux on the surface of the obstacle for velocity components and total salinity respectively, and vanishing of all perturbations at infinity,

\[
\begin{align*}
\mathbf{v} \bigg|_{z=0} &= v_1(x, z), \\
\mathbf{v} \bigg|_{z=h} &= s_1(x, z), \\
\mathbf{v}_s \bigg|_{z=h} &= 0, \\
\nabla \cdot \mathbf{v} \bigg|_{z=h} &= 0,
\end{align*}
\]

where \( U \) is the uniform free stream velocity at infinity, \( \mathbf{n} \) is external normal unit vector to the surface, \( \Sigma \), of the obstacle which can be either a plate or a wedge with length, \( L \), and height, \( h \), or, \( 2h \), \( P_1 \), \( v_1 \) and \( s_1 \) are initial perturbations of the fields under consideration which are generated by diffusion-induced flow due to interruption of the molecular transport of the stratifying agent by impermeable surface of the obstacle [14, 15].

It should be noticed that the instantaneous patterns of stratified flows are noticeably different from those of homogeneous fluid flows as it can be seen from a number of our previous papers [11, 14, 16], though some general processes of the flows evolution over time have a lot of similar features. But the present study is focused only on analysis of the stratified flow formation around a moving tilted plate, and a detailed comparison of results for the cases of stratified and homogeneous fluids the authors leave on future publications.

The set of equations and boundary conditions (1) – (2) are characterized by a number of parameters which contain length \( (\Lambda, L, h) \) or time \( (T_b, T_U = L/U) \) scales and characteristics of the body motion or dissipative coefficients.

Large dynamic scales which are internal wave length, \( \lambda = U T_b \), and viscous wave scale,

\[
\Lambda_v = \sqrt{g N / \Lambda} = \sqrt{\frac{\lambda}{\Lambda}} \left( \delta_N^v \right)^2,
\]

characterize the attached internal wave fields structure [13, 15].

The flow fine structure is characterized by universal microscales, \( \delta_N^v = \sqrt{v/N} \), \( \delta_N^s = \sqrt{\kappa_s / N} \), defined by the dissipative coefficients and buoyancy frequency, which are analogues of the Stokes scale on an oscillating surface, \( \delta_N^s = \sqrt{v/\omega} \) [15]. Another couple of parameters, such as Prandtl’s and Peclét’s scales, are determined by the dissipative coefficients and velocity of the body motion, \( \delta_U^v = v/U \) and \( \delta_U^s = \kappa_s / U \). The scale \( \delta_U^v = v/U \) characterizes flows of a homogeneous fluid too, as well.
Relations of the basic length scales produce dimensionless parameters such as Reynolds, \( \text{Re} = L/\delta_u = U/L\nu \), internal Froude, \( \text{Fr} = \lambda/2nL = U/\sqrt{\text{NL}} \), Pécel, \( \text{Pe} = L/\delta_p = U/L\kappa_S \), sharpness factor, \( \xi = L/h \) or fullness of form, \( \xi_S = S/Lh \), where \( S \) is the cross-sectional area of an obstacle, and, as well, relations specific for stratified media. The additional dimensionless parameters includes length scales ratio, \( C = \Lambda/L \), which is the analogue of reverse Atwood number, \( \text{At}^{-1} = (\rho_1 + \rho_2)/(\rho_1 - \rho_2) \), for a continuously stratified fluid.

Such a variety of length scales with their significant differences in values indicates complexity of internal structure even of such a slow flow generated by small buoyancy forces which arise due to the spatial non-uniformity of molecular flux of the stratifying agent.

The large length scales prescribe size selection for observation or calculation domains which should contain all the studied structural components, such as upstream perturbations, downstream wake, internal waves, vortices, while the microscales determine grid resolution and time step. At low velocities of the body motion, the Stokes scale is a critical one, while at higher velocities the Prandtl’s scale is dominant.

The system of equations (1) with the boundary conditions (2) is solved numerically using new solvers and dynamic mesh libraries of own development in frame of the computational package OpenFOAM with open source based on the finite volume method [16]. For discretization of the convective terms and the time derivative of the governing equations, a limited TVD-scheme and a second-order implicit asymmetric three-point scheme with backward differencing are used, respectively, which ensure minimal numerical diffusion, absence of non-physical oscillations of the solution, and a good time resolution of the physical process. For calculating the diffusion terms based on the Gauss theorem within orthogonal grid, surface normal gradient is evaluated at a cell face using a second order normal-to-face interpolation of the vector connecting the centers of the two neighbouring cells. In non-orthogonal grid regions, an iterative procedure with a user specified number of cycles is used for non-orthogonal correction of errors caused by a grid skewness.

Algorithm for construction of an orthogonal computational grid around a plate oriented at an arbitrary angle to horizon consists in creation of 4 main mesh blocks. They are an inner cylinder rotated together with the tilted plate when its angle position is changed, a mesh block merged to the cylinder and supplementing it to a parallelepiped, and two mesh blocks connected to the parallelepiped from its both sides (fig.1). When the tilted plate moves, the central parallelepiped mesh block is moved together with it, while the side mesh blocks are stretched and shortened, respectively, so that the inlet and outlet vertical boundaries are kept unmoved.

![Figure 1: Fragment of the computational grid around a tilted plate, \( L = 10 \text{ cm}, h = 0.5 \text{ cm}, \alpha = 20^\circ \).](image)

The spatial dimensions of computational cells were chosen from the condition of an adequate resolution of the finest flow components associated with the stratification and diffusion effects, which impose significant restrictions on the minimum spatial step. In high-gradient regions of the flow, at least several computational cells must fit the minimal linear scale of the problem. Calculation time step, \( \Delta t \), was defined by the Courant’s condition, \( \text{Co} = |v|\Delta t/\Delta r \leq 1 \), where \( \Delta r \) is the minimal size of grid cells and \( v \) is the local flow velocity. The general number of computational grid cells in the numerical model was about \( 2 \cdot 10^6 \). The calculations were partially performed using resources of the Supercomputing Center of Lomonosov Moscow State University [17].
The developed codes for stratified flows were used for 2D flows calculation in a homogeneous fluid [14]. The size of the grid cell in this case is prescribed by the Prandtl’s scale $\delta_p = v/\ell U$. In 3D case the system of equations for a homogeneous fluid becomes degenerated on singular perturbed components [15] and overdetermined.

### 3 Computation results

Stratified flows around obstacles can be classified into a number of typical flow regimes depending on prevailing structural components in the flow, such as vortices, which are common for all kinds of fluids, and internal waves, upstream perturbations, and high-gradient ligaments (thin shells, fibers, interfaces, etc.), which are specific for stratified media. The approach developed gives a possibility to study all the flow components mentioned in the frame of a single formulation based on the fundamental system of fluid mechanics equations by changing the fluid type through setting the stratification, viscosity and diffusion properties of the fluid.

Increase in velocity of plate movement leads to a series of structural transformations of stratified flow from multi-layered circulating diffusion-induced flows [11], which are formed in a quiescent stratified fluid without imposing any external mechanical disturbances, to unsteady vortex and fine-structural regimes when all the flow components are involved in a complex nonlinear interaction [14].

A great practical interest lies in visualization and analysis of instantaneous patterns of stratified flow at unsteady vortex regime when all the flow components, such as internal waves, vortices, flow fine structure (ligaments, fine interfaces, shells, fibers, etc.), are simultaneously formed at the leading edge of the plate being in active mutual interactions. At some parameters of the problem, e.g. large tilt angles of the plate, the flow structure is essentially unsteady that is manifested in active interactions of multi-scale vortices with each other, flow fine structure, and even attached internal waves which, in this case, substantially larger than the area of observation. At other parameters, e.g. small tilt angles, some of the flow components are more pronounced and able to preserve their primary forms for a longer time and drift downstream as a more or less stable chain of vortex structures.

Multiple mutual interactions of the flow components with different scales are manifested in the patterns of vorticity field at different tilt angles of the moving plate as illustrated in Fig. 1. For each value of tilt angle, 4 instantaneous images of the vortex flow are shown, which correspond to a set of distinguished flow regimes at the initial stage of flow evolution over time ($t_{\text{max}} = 5.0L/U_0$).

At small tilt angles of the moving plate ($\alpha < 3^\circ$), vortex structures are formed on the both sides of the plate around its leading edge with frequency, $f_v \approx 4 \text{ Hz}$, then separate from the surface at some distance downstream from the sharp edge and again reattach to it at the center of the plate (fig.2, a). Then, the vortices drift along the surface gradually losing their intensity as moving downstream. At the initial stage of the flow evolution, the wake flow has a typical vortex shedding structure in form of a sequence of vortex elements with alternately changing signs of vorticity with frequency, $f_w \approx 5 \text{ Hz}$. At time instant, $t \approx 1.5L/U_0$, when the vortices drift along the surface reach the trailing edge of the plate, an intensive interaction of the two vortex systems starts that leads to a loss of the initial vortex shedding structure. But already at time instant, $t \approx 2.0L/U_0$, a new regularity of the vortex flow is formed with frequency about two times less than the vortex shedding one. At small tilt angles of the plate, scales of the vortices in all the vortex systems distinguished are comparable to the plate’s thickness.

With increase in tilt angle of the moving plate ($5^\circ < \alpha < 10^\circ$), the vortex flow takes a significantly asymmetric pattern relative the plate’s plane (fig.2, b). The vortices drifting along the upper side of the plate with frequency, $f_v \approx 6 \text{ Hz}$, take more pronounce form and slightly increase in scales, while the vortex plane is located at a greater distance from the plate’s plane as compared to the case of small tilt angles. On the leeward side of the plate, vortex formation is mostly suppressed and the flow structure in this area takes a form of shear layer wave perturbations decaying downstream. Like in the previous case, the initial wake flow has a typical vortex shedding structure ($f_v \approx 5 \text{ Hz}$) and, after a merger of the two vortex systems, undergoes a stage of complex unsteady interaction taking a new regular structure at $t > 3.5L/U_0$ with oscillation frequency approx. two times less than the vortex shedding one.

At tilt angles of the moving plate varying in the range, $10^\circ < \alpha < 20^\circ$, the vortex flow past the windward side of the plate takes a much more intensive dynamics and a less regular structure as compared to all the previous cases considered, but at the same time perturbations on the leeward side of
the plate are fully suppressed by the undisturbed fluid (fig. 2, c). The leading-edge vortices do not already keep their identity but are involved in a more large-scale clockwise rotation, which is formed from merger of two linear vortex elements at the initial stage of the flow evolution or even from three ones at greater times, $t > 1.5L/U_0$. Average diameter of the vortex structures formed in the flow is not greater than the half of the vertical scale of the obstacle, $d < 0.5L \cdot \sin{\alpha}$, and frequency of the vortex formation is about $f \approx 4 \ Hz$. Initially, the wake flow structure has a typical vortex shedding form with the standard frequency, $f_w \approx 5 \ Hz$, but at $t > 1.8L/U_0$ it is determined by interaction of relatively large-scale vortices, which are formed over the windward side of the plate, and the wake vortex elements with scales comparable to the plate’s thickness.
Figure 2: Instantaneous patterns of vorticity field past a tilted plate uniformly moving in continuously stratified medium in successive time instants (images are placed from left to right as time increases), $U_0 = 100 \text{ cm/s}$, $N = 1.2 \text{ s}^{-1}$, $L = 10 \text{ cm}$, $h = 0.5 \text{ cm}$, $\alpha = 1^\circ, 5^\circ, 10^\circ, 20^\circ, 30^\circ, 45^\circ, 90^\circ$ (a-g) Blue and red colours are relevant to negative and positive signs of vorticity respectively.
Figure 3: Instantaneous patterns of horizontal component of density gradient field, $U_0 = 100$ cm/s, $N = 1.2$ s$^{-1}$, $L = 10$ cm, $h = 0.5$ cm, $\alpha = 1^\circ, 5^\circ, 10^\circ, 20^\circ, 45^\circ$ (a - e)

Blue and red colours are relevant to negative and positive values of the basic field (green) perturbations respectively.

Scales of the vortex flow arising past the windward side of the moving plate continue to grow with further increase in value of tilt angle of the plate ($20^\circ < \alpha < 30^\circ$). First 4 vortices generated by the leading edge of the plate are twisted into a clock wise rotating motion, center of the formed vortex structure being
gradually shifted to the plate’s trailing edge, thereby involving the vortex street in the wake into the rotating flow motion (fig.2, d).

This vortex structure is merged with another one formed from the next 4 vortices that increases scale of the structure up to the vertical size of the obstacle, \( d = L \cdot \sin(\alpha) \). Then it moves to the wake flow region and a new large-scale vortex structure rotating in counter clockwise direction is formed around the plate’s trailing edge with its external shell consisting of initial vortex elements of the vortex street. Duration of all the cycle of the wake flow formation is about \( t_{90}^L/U_0 \), which is then repeated in a more uniform form due to attenuation of inertial effects of instantaneous start of the plate movement. Thus, a typical flow vortex structure, formed at larger times over the windward side of the plate, consists of a pair of large and small same-sense vortices similar to dual leading-edge vortices observed experimentally for the cases of flapping dragonfly-like [18] and non-slender delta wings [19].

Development of the vortex flow with further increase in value of tilt angle of the moving plate (\( \alpha > 30^\circ \) ) is manifested in growing clarity and scale of the process of alternately forming two main vortex flow elements, such as a single vortex structure with clockwise rotation formed over the windward side of the plate and another one with counter clockwise rotation involving the primary vortex street (fig.2, e, f). In the extreme case of \( \alpha = 90^\circ \), two coherent large-scale vortices symmetric relative the plate’s central horizontal plane are formed, which have a spiral structure due to a twisting of the vortex element chains generated by the edges of the plate (fig.2, g).

Each physical variable solved by the governing system of equations (1) discloses some new important features of the stratified flow that contributes to a better understanding of the general physical processes involved. From horizontal component of density gradient field, \( \frac{\partial \rho}{\partial t} \), presented in Fig.3 for different tilt angles of the plate, one can get an additional information on flow fine structural elements which cannot be distinguished from other physical fields. In the present paper, a particular attention is paid to this field as it is in a linear dependence on light refraction ratio which is the parameter visualized in laboratory experiments by schlieren instruments. In the laboratory experiments conducted in the Institute for Problem in Mechanics of the RAS [13], a rich diversity of fine structural components was revealed around moving bodies in a stratified fluid using schlieren instruments, including fine-scale ligaments, interfaces, shells, fibers, etc., which are very similar to those observed in the patterns of horizontal component of density gradient field obtained by numerically solving the governing set of equations.

The patterns of the field consist of a variety of fine-scale multi-layered structures of both signs, which are mainly oriented along streamlines of the vortex flow elements forming systems of spiral curls typical for vortices. The fine structural elements are localized, as well, in flow regions where active interactions of different flow components with each other, the free stream and the rigid boundaries of the obstacle occur. The fineness of the horizontal component of density gradient field structure can be explained by the smallness of the ratio between the diffusion coefficient and the kinematic viscosity.

At relatively small tilt angles of the plate to horizon (fig.3, a–c), the shortest and the finest structures are gathered around the plate surface on vortex shells of the leading-edge vortices, while in the wake flow, the structures are longer and thicker with much more complicated shapes. Due to a growing intensity of interactions between the flow components with increase in tilt angle of the moving plate (fig.3, d, e), the field geometry is significantly complicated together with growth in number and expansion of scale range of the typical layered structures over the windward side of the plate. An important feature of the patterns of horizontal component of density gradient field at \( \alpha > 20^\circ \) is in formation of multilayered fine-scale structural elements in the direct vicinity of the plate surface due to intensive interactions of large- and small-scale vortex flow components with the rigid boundaries of the plate.

4 Conclusions

Instantaneous patterns of stratified flow structure around a uniformly moving tilted plate are studied numerically for the first time at unsteady vortex regime using solvers and libraries of own development of the OpenFOAM package based on the system of fundamental differential equations for incompressible viscous stratified fluid with no-slip and no-flux boundary conditions that allowed to distinguish principal stages of the flow formation over time and evaluate spatial and temporal scales of the vortex flow elements for a wide range of tilt angles of the plate to horizon.

The stratified flow is a combination of multiscale structure components, such as internal waves, vortices and ligaments (fine flow structures: interfaces, shells, fibres, etc.), which are simultaneously
formed at the leading edge of the plate and are in active mutual interactions. At small tilt angles of the plate, the leading-edge vortices preserve their primary forms and drift downstream as a more or less stable chain of vortex structures, while at larger tilt angles, the stratified flow structure is characterized by multiple interactions of multi-scale vortices between each other, with the free stream, the plate surface, and even the attached internal waves which, in this case, substantially larger than the area of observation.

The patterns of the horizontal component of density gradient field, which is in a linear dependence on light refraction ratio visualized in laboratory experiments by schlieren instruments, consist of a variety of fine-scale multi-layered structures of both signs, which are mainly oriented along streamlines of the vortex flow elements forming systems of spiral curls typical for vortices and localized, as well, in flow regions where active mutual interactions of different flow components with the free stream and the rigid boundaries of the plate occur.

The developed approach is universal and has a resource to be extended for investigation of 3D stratified flows in a wide range of geometrical, dynamical and physical parameters of incompressible fluid, compressible gas and plasma flows. In 2D case, similar calculations (with loss of buoyancy and diffusion effects) can be performed for viscous homogeneous fluid flows, as well.

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References


