THEORETICAL APPROACH OF THE MEAN HYDRODYNAMIC FIELD IN THE CONICAL TAYLOR-COUETTE FLOW

F. Yahi1, 2, A. Bouabdallah1, F. Rousset3, D. Henry4, T. Adachi5, V. Botton4, Y. Hamnoune1

1 Thermodynamics and Energetic Systems Laboratory, Faculty of Physics, University of Sciences and Technology Houari Boumediene, B.P. 32 El Alia 16111 Bab Ezzouar, Algiers, Algeria.
2 Genie Physical of Hydrocarbons Laboratory, Faculty of hydrocarbons and chemistry, University M’Hamed Bougara, 35000 Boumerdes, Algeria.
3 CETHIL, Université de Lyon, INSA de Lyon/Université Lyon 1, INSA, Bâtiment Sadi Carnot, 9 rue de la Physique, 69621 Villeurbanne Cedex, France.
4 Laboratoire de Mécanique des Fluides et d’Acoustique, CNRS/ Université de Lyon, Ecole Centrale de Lyon/Université Lyon 1/INSA de Lyon, ECL, 36 Avenue Guy de Collongue, 69134 Ecully Cedex, France.
5 Department of Mechanical Engineering, Akita University, Tegata-Gakuen 1-1, Akita 010-8502, Japan.

Abstract

The flow between rotating cones have been investigated experimentally and numerically by several authors. There are a few analytical studies concerning the so called flow system. The flow is defined by an incompressible viscous fluid characterized by constant physical properties (density and kinematic viscosity) between two coaxial cones. The cones have the same apex angle, giving a constant radial gap. The inner cone rotates with an angular velocity $\Omega$ and the outer one is maintained at rest. Furthermore, the basic flow between rotating coaxial cones is fully three-dimensional resulting from the balance between centrifugal and viscous forces. The present work focuses in analytical and numerical approach to establish the mean velocity profiles characterizing the basic flow. For that purpose, a particular curvilinear coordinate system is used to establish the governing equation corresponding to the conical Taylor-Couette flow system. The obtained results indicate that the flow is dominated mainly by the tangential velocity component.

Keywords: coaxial cones, laminar-turbulent transition, spiral mode, finite volume method, Taylor vortex.

1 Introduction

The conical Taylor-Couette flow has been studied numerically and experimentally by numerous author. M. Wimmer [1-3] was the first author who has highlighted the installation of Taylor vortices in the flow between rotating cones and showed that the flow is sensitive to the initial and boundary conditions, the gap size, the rotation velocity and the apex angle. M.N. Noui-Mehidi et al [4] have examined the laminar-turbulent transition in the case of small gap configuration and showed that the flow develops from the laminar regime towards helical motion through the formation of Taylor vortices by varying the rotation speed of the inner cone. Recently, Xio F. et al [5] showed in their numerical simulations that maximum velocity and pressure magnitudes decrease with increasing the cone inclination. Li Qiu-shu et al studied the transition to Taylor vortices between rotating conical cylinders in the case of rigid boundary, where both end plates are stationary[6].

The flow between two cones was widely considered for understanding the centrifuge performance used in various industrial sectors like pharmaceutical and food. The basic flow between rotating coaxial cones is fully three-dimensional resulting from the balance between centrifugal and viscous forces.

In the present study, we have used a new curvilinear coordinate system in order to get constant boundary conditions. Consequently, we have established the expression of different operators giving the governing equations of the conical Taylor-Couette flow.
2 Coordinate system

To describe the flow between two cones we cannot use neither the cylindrical nor the spherical coordinate system in the raison of the radius variation versus the axial direction (Fig.1). Furthermore, the boundary conditions associated with the cone system are not constant along the generatrix. For this reason, we choose an orthogonal curvilinear coordinates system which consists of three axes X, θ, and Z, namely, Z is supported by the generatrix of the inner cone. Thus, X is the coordinate axis perpendicular to Z axis and θ is the azimuthal coordinate. Therefore, the new coordinates are, x, θ and z (Figure 1). The Cartesian coordinates (X, Y, Z) are defined in the curvilinear coordinate system by the following expressions:

\[ X = r \cos \theta, \ Y = r \sin \theta, \ Z = -x \sin \varphi + z \cos \varphi \] with \( r = x \cos \varphi + z \sin \varphi \).

![Figure 1: Coordinates system](image)

3 Governing equation in conical Taylor-Couette flow system

We consider an incompressible viscous fluid flow filling the gap between two coaxial cones; the inner cone is rotating with an angular velocity \( \Omega_1 \) and the outer cone is fixed (\( \Omega_2 = 0 \)). The cones constituting the flow system have the same apex angle \( \varphi = 12 \) given a constant annular space \( \delta \). The working fluid is characterized by constant physical properties: density \( \rho \), kinematic viscosity \( \nu \). The governing equations characterizing the considered flow are the Navier-Stokes equations.

\[
\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = -\frac{1}{\rho} \nabla p + \nu \Delta \mathbf{V} + f \tag{1}
\]

\[
\Delta \mathbf{V} = 0 \tag{2}
\]

The projection of these equations (1 and 2) in the curvilinear coordinate system described above is shown by the following expressions:

\[
\frac{\partial V_x}{\partial t} + V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} + V_z \frac{\partial V_x}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial V_y}{\partial y} \tag{3}
\]

\[
\frac{\partial V_y}{\partial t} + V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y} + V_z \frac{\partial V_y}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - \frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\partial V_x}{\partial x} \tag{4}
\]
\[
\begin{align*}
\frac{\partial V_x}{\partial t} + V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} - V_z \frac{\partial V_x}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \\
+ \nu \left( \frac{\partial^2 V_x}{\partial y^2} + \frac{\partial^2 V_x}{\partial z^2} \right) - g \cos \phi 
\end{align*}
\] 
(5)

**Continuity equation**

\[
\frac{\partial V_x}{\partial x} + \frac{1}{r} \frac{\partial V_x}{\partial \theta} + \frac{\partial V_x}{\partial z} + \frac{V_x}{r} \frac{\partial \phi}{\partial \theta} = 0
\]
(6)

**Boundary conditions**

The associated boundary conditions related to the studied problem are:

\[ x = 0 \quad V_x = V_z = 0 \quad V_y = \Omega r \quad \text{and} \quad x = d \quad V_x = V_z = V_y = 0 \]

In order to simplify the problem and reducing the unknown's numbers, the governing equations are written in dimensionless form using the following reduced variables and functions given by:

**Velocity field:** \[ V_x^* = V_x / V_0 \quad \text{with} \quad V_0^* = (V_x^*, V_y^*, V_z^*) \quad \text{and} \quad V_0 = \Omega R_{\text{max}} \]

**Pressure:** \[ p^* = p / (\rho V_0^2) \]

**Time and space variables:**

*Time:* \[ \tau = \frac{V_x}{d} \quad \text{and} \quad \frac{\partial}{\partial \tau} = \frac{\partial}{\partial t} = \frac{V_x}{d} \frac{\partial}{\partial \tau} \quad \text{with} \quad V_x \quad \text{is the kinematic viscosity} \]

*Space:* \[ \eta = \frac{x}{d}, r^* = \frac{r}{d}, \zeta = \frac{z}{d} \quad \text{with} \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial \eta} \frac{\partial}{\partial \eta}, \quad r^* = \zeta \sin \varphi + \eta \cos \varphi \quad \text{and} \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial \zeta} \frac{\partial}{\partial \zeta} = \frac{1}{d} \frac{\partial}{\partial \zeta} \]

Thenceforth, we introduce the control parameters and the dimensionless variables, in order to reduce the number of variables:

**Reynolds number:** \[ R_e = \frac{V_x d}{\nu} \quad \text{and} \quad \text{Froude number:} \quad F_r = \frac{V_x}{\sqrt{gd}} \]

**Hydrodynamic equations**

\[
\begin{align*}
\frac{1}{\text{Re}} \left( \frac{\partial V_x^*}{\partial \tau} + V_x^* \frac{\partial V_x^*}{\partial \eta} + \frac{V_x^*}{r} \frac{\partial V_x^*}{\partial \theta} + \frac{V_x^*}{r^2} \frac{\partial V_x^*}{\partial \zeta} \right) &= \frac{\partial p^*}{\partial \eta} \\
+ \frac{1}{\text{Re}} \left( \frac{\partial V_y^*}{\partial \tau} + V_x^* \frac{\partial V_y^*}{\partial \eta} + \frac{V_y^*}{r} \frac{\partial V_y^*}{\partial \theta} + \frac{V_y^*}{r^2} \frac{\partial V_y^*}{\partial \zeta} \right) &= \frac{1}{F_r^2} \sin \varphi \\
+ \frac{1}{\text{Re}} \left( \frac{\partial V_z^*}{\partial \tau} + V_x^* \frac{\partial V_z^*}{\partial \eta} + \frac{V_z^*}{r} \frac{\partial V_z^*}{\partial \theta} + \frac{V_z^*}{r^2} \frac{\partial V_z^*}{\partial \zeta} \right) &= \frac{1}{F_r^2} \cos \varphi
\end{align*}
\] 
(7)

\[
\begin{align*}
\frac{1}{\text{Re}} \left( \frac{\partial V_x^*}{\partial \tau} + V_x^* \frac{\partial V_x^*}{\partial \eta} + \frac{V_x^*}{r} \frac{\partial V_x^*}{\partial \theta} + \frac{V_x^*}{r^2} \frac{\partial V_x^*}{\partial \zeta} \right) &= \frac{1}{F_r^2} \sin \varphi \\
+ \frac{1}{\text{Re}} \left( \frac{\partial V_y^*}{\partial \tau} + V_x^* \frac{\partial V_y^*}{\partial \eta} + \frac{V_y^*}{r} \frac{\partial V_y^*}{\partial \theta} + \frac{V_y^*}{r^2} \frac{\partial V_y^*}{\partial \zeta} \right) &= \frac{1}{F_r^2} \cos \varphi \\
+ \frac{1}{\text{Re}} \left( \frac{\partial V_z^*}{\partial \tau} + V_x^* \frac{\partial V_z^*}{\partial \eta} + \frac{V_z^*}{r} \frac{\partial V_z^*}{\partial \theta} + \frac{V_z^*}{r^2} \frac{\partial V_z^*}{\partial \zeta} \right) &= \frac{1}{F_r^2} \sin \varphi
\end{align*}
\] 
(8)

\[
\begin{align*}
\frac{1}{\text{Re}} \left( \frac{\partial V_x^*}{\partial \tau} + V_x^* \frac{\partial V_x^*}{\partial \eta} + \frac{V_x^*}{r} \frac{\partial V_x^*}{\partial \theta} + \frac{V_x^*}{r^2} \frac{\partial V_x^*}{\partial \zeta} \right) &= \frac{1}{F_r^2} \sin \varphi \\
+ \frac{1}{\text{Re}} \left( \frac{\partial V_y^*}{\partial \tau} + V_x^* \frac{\partial V_y^*}{\partial \eta} + \frac{V_y^*}{r} \frac{\partial V_y^*}{\partial \theta} + \frac{V_y^*}{r^2} \frac{\partial V_y^*}{\partial \zeta} \right) &= \frac{1}{F_r^2} \cos \varphi \\
+ \frac{1}{\text{Re}} \left( \frac{\partial V_z^*}{\partial \tau} + V_x^* \frac{\partial V_z^*}{\partial \eta} + \frac{V_z^*}{r} \frac{\partial V_z^*}{\partial \theta} + \frac{V_z^*}{r^2} \frac{\partial V_z^*}{\partial \zeta} \right) &= \frac{1}{F_r^2} \sin \varphi
\end{align*}
\] 
(9)

**Continuity equation**

\[
\frac{\partial V_x^*}{\partial \eta} + \frac{1}{r} \frac{\partial V_x^*}{\partial \theta} + \frac{V_x^*}{r} \frac{\partial \phi}{\partial \theta} = \frac{V_x^*}{r} \sin \varphi
\]
(10)
Boundary conditions: \( \eta = 0 \quad V_\eta^* = V_\eta^* = 0 \quad V_\theta^* = 1 \) and for \( \eta = 1 \quad V_\eta^* = V_\zeta^* = V_\theta^* = 0 \)

In order to separate the mean hydrodynamic field of the disturbed hydrodynamic field, we adopted the following notations:

\[ V_\eta^* = V_\eta + v_\eta, \quad V_\theta^* = (V_\theta + v_\theta), \quad V_\zeta^* = (V_\zeta + v_\zeta), \quad \rho^* = (\bar{\rho} + \rho) \]

### Mean hydrodynamic field

\[
\frac{1}{\Re} \frac{\partial V_\eta}{\partial \eta} + \frac{1}{r} \frac{\partial V_\theta}{\partial \theta} + \frac{1}{r^2} \frac{\partial V_\zeta}{\partial \zeta} + \frac{V_\eta^2}{r} \cos \phi + \frac{V_\zeta}{r} \sin \phi = -\frac{\partial \bar{P}}{\partial \eta} + \frac{1}{\Re} \left( \frac{1}{r^2} \frac{\partial^2 V_\eta}{\partial \theta^2} + \frac{1}{r \sin \phi} \frac{\partial V_\theta}{\partial \theta} + \frac{1}{r^2} \frac{\partial^2 V_\zeta}{\partial \zeta^2} + \frac{2 \cos \phi \frac{\partial V_\eta}{\partial \eta} + \sin \phi \frac{\partial V_\zeta}{\partial \zeta}}{r} \right) + \frac{1}{Fr} \sin \phi
\]

\[
\frac{1}{\Re} \frac{\partial V_\theta}{\partial \eta} + \frac{1}{r} \frac{\partial V_\zeta}{\partial \theta} + \frac{1}{r^2} \frac{\partial^2 V_\eta}{\partial \zeta^2} + \frac{V_\theta}{r^2} \cos \phi + \frac{1}{r^2} \frac{\partial V_\zeta}{\partial \eta} = -\frac{1}{Fr} \frac{\partial \bar{P}}{\partial \theta}
\]

\[
\frac{1}{\Re} \left( \frac{1}{r^2} \frac{\partial^2 V_\eta}{\partial \theta^2} + \frac{\sin \phi}{r} \frac{\partial V_\theta}{\partial \theta} + \frac{1}{r^2} \frac{\partial^2 V_\zeta}{\partial \eta^2} + \frac{2 \sin \phi \frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_\zeta}{\partial \eta} \cos \phi + \frac{V_\zeta}{r} \sin \phi}{r^3} \right) = \frac{1}{Fr^2} \cos \phi
\]

\[
\frac{\partial V_\theta}{\partial \eta} + \frac{1}{r^2} \frac{\partial V_\zeta}{\partial \theta} + \frac{V_\eta}{r^2} \cos \phi + \frac{V_\zeta}{r} \sin \phi = 0
\]

**Boundary conditions:** \( \eta = 0 \quad V_\eta = V_\zeta = 0 \quad V_\theta = 1 \) and for \( \eta = 1 \quad V_\eta = V_\zeta = V_\theta = 0 \)

### 4. Hypotheses and simplifications

View the problem complexity expressed in its general formulation, related to the three-dimensional flow character. We are led to propose some simplifying assumptions, namely:

Steady laminar flow: \( R_\eta < R_\zeta \)

Stationary flow regime: \( \partial / \partial t = 0 \)

The problem is axisymmetric: \( \partial / \partial \theta = 0 \)

Small annular gap configuration: \( 0 \leq \delta \leq 1 \)

\[ 0 \leq \eta \leq 11.9 \leq \zeta \leq 43 \] with \( 0 \leq \eta \cotan \phi < 0.1 \) so \( \frac{\eta \cotan \phi}{\zeta} < 1 \)

whence, \( r^* = \zeta \sin \phi \left( 1 + \frac{\eta}{\zeta} \cotan \phi \right) = \zeta \sin \phi \)

The mean field is characterized by the following three components: \( V = (V_\eta, V_\theta, V_\zeta) \)

such as: \( V_\eta = V_\eta(\eta, \zeta) \), \( V_\theta = V_\theta(\eta, \zeta) \), \( V_\zeta = V_\zeta(\eta, \zeta) \)

These assumptions are insufficient. However, the boundary layer theory must be used to simplify the complexity of the problem.

\( \zeta = 0(1) \), \( \eta = 0(\varepsilon) \), \( V_\eta = 0(1) \), \( V_\theta = 0(1) \), \( V_\zeta = 0(1) \)

As a first approximation, we can neglect all terms of order \( \varepsilon \) in the equations system to the unit content in the Left members, and the right of members of the terms of order \( 1 / \varepsilon \) and are neglected before those of order \( 1 / \varepsilon^2 \).

\[
-\frac{V_\zeta^2}{\zeta \sin \phi} \cos \phi = -\frac{\partial \bar{P}}{\partial \eta} + \frac{1}{\Re} \left( \frac{1}{r^2} \frac{\partial^2 V_\eta}{\partial \theta^2} - \frac{1}{\zeta \sin \phi} \frac{\partial V_\theta}{\partial \theta} - \frac{1}{r^2} \frac{\partial^2 V_\zeta}{\partial \eta^2} \right) + \frac{1}{Fr^2} \sin \phi
\]
\[ \nabla_n \partial V_n + \nabla_\xi V_\xi \cot \eta + V_\xi V_n + \nabla_\eta V_\eta = \frac{1}{\text{Re}} \nabla_\eta V_\eta \] (16)

\[ \nabla_n \partial V_n - \nabla_\xi V \xi = - \frac{\partial \tilde{P}}{\partial \xi} + \frac{1}{\text{Re}} \nabla_\eta V_\eta \frac{1}{Fr} \cos \phi \] (17)

\[ \nabla_\eta \partial V_n + \nabla_\xi V_\xi \cos \phi + \nabla_\xi \sin \phi \nabla_\eta V_\eta + \frac{1}{\xi} V_\xi = 0 \] (18)

Boundary conditions: \( \eta = 0 \quad \nabla_\eta = \nabla_\xi = 0 \quad \nabla_\eta = \frac{\Omega \xi \text{d} \sin \phi}{V_\eta} \quad \text{and for} \quad \eta = 1 \quad \nabla_\eta = \nabla_\xi = \nabla_\eta = 0 \)

5. SOLVING PROBLEM

By using the cross-derivation of equations (15) and (17), the pressure terms have been eliminated and the system becomes:

\[ \nabla_n \partial V_n + 2V_n \nabla_\eta V_\eta = \frac{1}{\text{Re}} \nabla_\eta V_\eta \] (19)

\[ \nabla_n V_\eta + V_\xi V_n + V_\xi \partial V_\eta + V_\xi = \frac{1}{\text{Re}} \nabla_\eta V_\eta \] (20)

\[ \nabla_\eta V_\eta + \frac{\partial V_\xi}{\partial \xi} + \frac{1}{\xi} V_\xi = 0 \] (21)

Boundary conditions: \( \eta = 0 \quad \nabla_\eta = \nabla_\xi = 0 \quad \nabla_\eta = \frac{\Omega \xi \text{d} \sin \phi}{V_\eta} \quad \text{and for} \quad \eta = 1 \quad \nabla_\eta = \nabla_\xi = \nabla_\eta = 0 \)

In order to solve the previous equation system, we have imposed the hydrodynamic field solution in the affine transformation form. The proposed solution obeys to the hydrodynamic boundary layer principle.

\( \nabla_\eta = F(\eta), \quad \nabla_\xi = g(\xi)G(\eta), \quad \nabla_\xi = h(\xi)H(\eta). \) Assuming that, \( g(\xi) = h(\xi) = \xi \)

\[ F'(\eta)H'(\eta) + F(\eta)H''(\eta) - 2G(\eta)G'(\eta) = \frac{1}{\text{Re}} H''(\eta) \] (22)

\[ F(\eta)G'(\eta) + 2H(\eta)G(\eta) = \frac{1}{\text{Re}} G''(\eta) \] (23)

\[ F'(\eta) + 2H(\eta) = 0 \] (24)

Boundary conditions:

\( \eta = 0 \quad F(\eta) = H(\eta) = 0 \quad G(\eta) = \frac{\Omega \xi \text{d} \sin \phi}{V_\eta} = 1 \quad \text{and} \quad \eta = 1 \quad F(\eta) = H(\eta) = G(\eta) = 0 \)

And assuming that, \( V_\eta = \Omega d \sin \phi \), we obtained:

Boundary conditions: \( \eta = 0 \quad F(\eta) = H(\eta) = 0 \quad G(\eta) = 1 \quad \text{and} \quad \eta = 1 \quad F(\eta) = H(\eta) = G(\eta) = 0 \)

The previous equation system has been solved by using the explicit Runge-Kutta method with a constant sampling. The obtained solutions are calculated for a number of points \( n = 100 \) with an accuracy of the order of \( 10^{-25} \) corresponding to a theoretical Reynolds number \( Re_{th} = 1 \).

Numerical method:

In order to confirm the obtained results, we attempted to perform a numerical approach based on finite volumes and using the Fluent computing software. All results are compared with those obtained previously and with those obtained by using a spectral element method associated with graphic elements.
As for treatment for trapezoidal geometry, we used a mapping from trapezoidal geometry to square geometry \[8\].

6. RESULTS AND DISCUSSION

It is assumed that the radial component of the basic flow depends only on the radial position \(\eta\) and does not depend on the axial position \(\zeta\). The obtained solution of the function \(F(\eta)\) is Polynomial of order 5 whose coefficients are summarized in Table 1: 

\[
F(\eta) = f_1\eta + f_2\eta^2 + f_3\eta^3 + f_4\eta^4 + f_5\eta^5.
\]

It has been assumed previously that the tangential component of the velocity is linearly proportional to the axial position \(\zeta\) and is defined as the product of the function \(G(\eta)\) by \(g(\zeta) = \zeta\).

Similarly, the axial velocity component also depends on the axial position \(\zeta\) and evolves as the product of the function \(h(\zeta) = \zeta\) and \(H(\eta)\).

The analysis of the obtained results showed us that the function \(F(\eta)\) (dimensionless radial component) passes through a minimum \(F_{\text{min}} = 5.226 \times 10^{-3}\) corresponding to the radial position \(\eta = 0.4751719640\) \(\approx 0.48\) Fig. 2-a. Whereas, the function \(H(\eta)\) (dimensionless axial component) vanishes for three radial positions \(\eta = 0, \eta = 1\) and \(\eta = 0.4751719640=0.48\). \(\eta = 0.4751719640\approx 0.48\) corresponds to the position where the radial velocity becomes minimum.

The dimensionless axial component passes through two extreme points or one is a maximum at \((\eta = 0.19063, 0.0085)\) and the other is a minimum \((\eta = 0.769, -0.007617)\).

Analyzing the amplitudes of the two axial and radial velocity components with respect to the tangential component, it was found that \(G_{\text{max}}(\eta) = 200F_{\text{max}}(\eta)\) and \(G_{\text{max}}(\eta) \equiv 118H_{\text{max}}(\eta)\), therefore \(H_{\text{max}}(\eta) \equiv 1.8F_{\text{max}}(\eta)\). This means that the flow is dominated by the tangential component of the velocity. These results are in good agreement with those obtained experimentally by Wimmer \[3\].

<table>
<thead>
<tr>
<th>Coefs.</th>
<th>Estimation</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_1)</td>
<td>(-5.8\times10^{-6})</td>
<td>(4.87\times10^{-7})</td>
</tr>
<tr>
<td>(f_2)</td>
<td>(-0.01)</td>
<td>(4.01\times10^{-6})</td>
</tr>
<tr>
<td>(f_3)</td>
<td>(0.23)</td>
<td>(1.14\times10^{-5})</td>
</tr>
<tr>
<td>(f_4)</td>
<td>(-0.17)</td>
<td>(1.33\times10^{-5})</td>
</tr>
<tr>
<td>(f_5)</td>
<td>(0.03)</td>
<td>(5.5\times10^{-6})</td>
</tr>
<tr>
<td>(f_6)</td>
<td>(-1.00)</td>
<td>(1.54\times10^{-5})</td>
</tr>
<tr>
<td>(f_7)</td>
<td>(5.24\times10^{-6})</td>
<td>(3.06\times10^{-7})</td>
</tr>
<tr>
<td>(f_8)</td>
<td>(0.03)</td>
<td>(2.32\times10^{-6})</td>
</tr>
<tr>
<td>(f_9)</td>
<td>(-0.07)</td>
<td>(8.81\times10^{-6})</td>
</tr>
<tr>
<td>(f_{10})</td>
<td>(0.06)</td>
<td>(1.85\times10^{-5})</td>
</tr>
<tr>
<td>(f_{11})</td>
<td>(-0.02)</td>
<td>(-2.18\times10^{-5})</td>
</tr>
<tr>
<td>(f_{12})</td>
<td>(0.003)</td>
<td>(1.35\times10^{-5})</td>
</tr>
<tr>
<td>(f_{13})</td>
<td>(1.0\times10^{-4})</td>
<td>(3.4\times10^{-6})</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coefs.</th>
<th>Estimation</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_1)</td>
<td>(0.1)</td>
<td>(1.95\times10^{-5})</td>
</tr>
<tr>
<td>(f_2)</td>
<td>(-0.35)</td>
<td>(3.87\times10^{-7})</td>
</tr>
<tr>
<td>(f_3)</td>
<td>(0.33)</td>
<td>(2.937\times10^{-6})</td>
</tr>
<tr>
<td>(f_4)</td>
<td>(-0.08)</td>
<td>(1.177\times10^{-7})</td>
</tr>
<tr>
<td>(f_5)</td>
<td>(-5.45 \times 10^{-4})</td>
<td>(2.347 \times 10^{-7})</td>
</tr>
<tr>
<td>(f_6)</td>
<td>(4.2 \times 10^{-4})</td>
<td>(2.76 \times 10^{-5})</td>
</tr>
<tr>
<td>(f_7)</td>
<td>(2.4 \times 10^{-4})</td>
<td>(1.77 \times 10^{-6})</td>
</tr>
<tr>
<td>(f_8)</td>
<td>(-1.66 \times 10^{-4})</td>
<td>(4.37 \times 10^{-6})</td>
</tr>
</tbody>
</table>

Table 1: Functions coefficient

Figure 2: a) Tangential velocity profile, b) Axial velocity profile c) Radial velocity profile
Numerical results:

By plotting the evolution of the axial velocity versus the axial position $z$, it has been found that it evolves according to a linear law of positive slope which makes the assumptions of the mathematical model very satisfactory (Figure 3). Consequently, the tangential velocity passes through a maximum in the vicinity of the inner cone and decreases until it reaches a zero value near the outer cone. The latter, greatly changes behavior law by going towards the weakest rays.

While the radial velocity passes through two limit positions $\eta = 0$ and $\eta = 1$ and reaches a minimum for the same position where the axial velocity $\eta = 0.478969 \approx 0.48$ is zero.
The numerical approach used confirms the hypothesis and the results obtained by the analytical calculation. We have obtained the same velocity profiles.

7. Conclusion

In the present study, it was found using a new coordinates system that it is possible to solve the equations of motion in the approximation of the small annular gap configuration in the rotating conical flow system. It was obtained that the flow is dominated by the tangential component of the velocity. Therefore, it has been found that the axial velocity evolves according to a linear law of positive slope versus z direction and makes the assumptions of the mathematical model very satisfactory. The obtained results are similar to those obtained in the case of classical Taylor-Couette Flow.

References