Abstract

The paper deals with the numerical simulation of the transonic flow through a mid-span turbine blade cascade by means of an in-house code based on the EARSM turbulence model of Hellsten [1] completed by the algebraic transition model of Straka and Přihoda [2]. The simulation using the transition model of Langtry and Menter [3] and Menter et al. [4] implemented in the commercial code ANSYS Fluent was used for the comparison. Simulations were carried out for the transonic regime close to the nominal regime. The flow separation on the suction side of the blade is caused by the interaction of the reflected shock wave with the boundary layer. The attention was focused on the modelling of the transition in the separated flow especially on the modelling of the length of the transition region. Numerical results were compared with experimental results.

Keywords: mid-section blade cascade, boundary layer transition, shock wave interaction

1 Introduction

An adequate modelling of the separation-induced transition is very important for the simulation of transonic flows through the blade cascades because the interaction of the shock waves with shear layers often causes the flow separation followed by transition. The flow separation and the separation-induced transition are caused by the adverse pressure gradient and/or by the interaction of the shock wave with the boundary layer in transonic flow regimes. A majority of experimental and numerical results concerns the separation bubble caused by the adverse pressure gradient on the upper side of aerofoils and existing correlations for the transition onset and length are based on these experimental data.

A long-term attention is paid to the problem of the shock-wave/boundary layer interaction in supersonic flow both in aeronautics and in turbomachinery. The incident-reflecting shock is the most common type of the shock-wave/boundary layer interaction in turbomachinery. Unfortunately, the majority of papers deal with the problem of the shock-wave interaction with either laminar or turbulent boundary layers. A detailed description of the shock-wave/boundary-layer interaction was given by Délery and Dussauge [5]. The flow pattern of the supersonic interaction depends both on the incident shock wave and the boundary-layer character. The recent progress in the shockwave/boundary-layer interaction from the experimental and numerical point of view is thoroughly described by Gaitonde [6]. The effect of the upstream boundary layer on the unsteadiness of shock-induced separation was described by Ganapathisubramani et al. [7]. The problem of the transition induced by shock-wave/boundary-layer interaction was recently solved in the TFAST project „Transition Location Effect on Shock Wave Induced Separation”. Besides experimental studies, see e.g. Flaszynski et al. [8], there are also some attempts to simulate separation-induced transition due to the shock wave/boundary-layer interaction, see e.g. Teramoto [9] and Luxa et al. [10].

This contribution deals with the numerical simulation of 2D compressible flow through the linear turbine blade cascade TR-Z-1 for the isentropic Reynolds number $Re_{2is} = 1.66 \times 10^6$ and for the isentropic Mach numbers $M_{2is} = 1.205$. The investigated blade cascade with the relative spacing $t/c = 0.718$ represents the mid-section of a 48” long rotor blade applied at the last stage of a large output steam turbine. Predictions were carried out by the in-house numerical code using the EARSM turbulence model completed by the algebraic bypass transition model and by the commercial code ANSYS Fluent using the SST turbulence model with $\gamma Re$ and $\gamma$ transition models.
2 Mathematical models

The mathematical model of compressible flow is based on conditionally-averaged Navier-Stokes equations completed by the constitutional relations for the ideal gas, the explicit algebraic turbulence model (EARSM) by Hellsten [1], the algebraic transition model proposed by Straka and Přihoda [2] and by the model of turbulent heat transfer based on the generalized gradient hypothesis according to Launder [11]. The EARSM model was modified into the form corresponding to models with the turbulent viscosity. The turbulent stress is given by the relation

\[ \tau_{ij} = \mu \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial U_k}{\partial x_k} \right) \right. \left. - \frac{2}{3} \delta_{ij} \rho k - \alpha' \rho k \right] \tag{1} \]

where the so-called extra-anisotropy tensor \( \alpha' \) is approximated by the polynomial of the 4th order with coefficients dependent on strain-rate and vorticity tensors and their invariants. Turbulent scales are expressed by the turbulent energy \( k \) and by the specific dissipation rate \( \omega \) by means of the SST turbulence model of Menter [12]. Turbulent time scale \( \tau \sim 1/\omega \) is replaced near the wall by the Kolmogorov viscous time scale. The production term in the turbulent energy equation was modified for the reduction of the undesirable over-production of the turbulent energy in the stagnation region (see Straka and Přihoda [13]). Production and destruction terms in the \( k \)-equation are multiplied by the intermittency coefficient \( \gamma \). Similarly, the effective viscosity is given by \( \mu_{eff} = \mu + \gamma \mu_{t} \) in the transition region. The generalized gradient hypothesis is used for modelling of the turbulent heat transfer in the form

\[ q_i = -C_i \frac{\tau_{ij}}{\omega} \frac{\partial T}{\partial x_j} \quad C_i = 0.3 \tag{2} \]

The transition model is based on the concept of different values of the intermittency coefficient in the boundary layer \( \gamma_i \) and in the free stream \( \gamma_e \). The intermittency coefficient in the boundary layer \( \gamma_i \) is expressed by the relation

\[ \gamma_i = 1 - \exp \left[ -\hat{n}\sigma (Re_i - Re_{th})^3 \right] \tag{3} \]

The transition onset is given by the empirical correlation for the momentum Reynolds number \( Re_{th} = f(Tu, x) \) where \( Tu \) is the free-stream turbulence level and \( x \) is the pressure-gradient parameter. The length of the transition region is expressed using the parameter \( N = \hat{n}\sigma Re_{th}^3 \) where \( \hat{n} \) is the spot generation rate and \( \sigma \) is the spot propagation rate introduced by Narasimha [14]. The parameter \( N \) for the attached flow is correlated by the empirical relation

\[ N = 0.86 \times 10^3 \exp \left( 2.134 \lambda \ln Tu - 59.23 \lambda - 0.564 \ln Tu \right) \quad \text{for} \quad \lambda \leq 0 \]

\[ N = 0.86 \times 10^3 Tu^{0.364} \exp \left( -10\sqrt{\lambda} \right) \quad \text{for} \quad \lambda > 0 \tag{4} \]

proposed by Solomon et al. [15]. The onset of transition in separated flow is given by the correlation according to Mayle [16] in the form

\[ Re_{th} = 300 Re_{th}^{0.7} + Re_{th} \tag{5} \]

where \( Re_{th} \) is the momentum Reynolds number at the separation and \( Re_{th} \) is the Reynolds number related to the distance of the separation from the leading edge. The transition length of the short separation bubble is given according to Walker [17] by the relation

\[ \hat{n}\sigma = \frac{Re_{th}^{1.34}}{40} \tag{6} \]

and by Mayle [16] by the relation

\[ \hat{n}\sigma = \frac{2.28 \times 10^4}{Re_{th}^{1.3}} \tag{7} \]

and so the same approach can be applied as in the attached flow.

For prediction of transitional flows in complex geometries, the application of local variables is necessary. Therefore the momentum Reynolds number is replaced by the maximum of the vorticity Reynolds number according to Langtry and Menter [3]. The vorticity Reynolds number is given by the relation

\[ Re_{\Omega} = \gamma^2 |\Omega| / \nu \tag{8} \]
where \( y \) is the distance from the wall and \( \Omega \) is the absolute value of the vorticity tensor. The link between both Reynolds numbers is expressed by the relation \( \text{Re}_\theta = \text{Re}_{\Delta \text{max}} / C \) where the parameter \( C \) depends on the pressure gradient expressed by the parameter

\[
L = \text{Re}_{\Delta}^2 \frac{\nu}{U_\infty^2} \frac{dU_r}{dx}.
\]  

(9)

The variation of the parameter \( C \) with the pressure gradient parameter was estimated by means of similar solutions of Falkner-Skan velocity profiles.

The system of governing equations has been implemented into the in-house numerical code. The code is based on the finite volume method of the cell-centered type with the Osher’s-Solomon’s approximation of the Riemann’s solver and the two-dimensional linear reconstruction with the Van Albada’s limiter. The governing equations are discretized using a multi-block quadrilateral structured grid with a block overlapping implementation. The algebraic transition model is described in detail by Fürst et al. [18].

The correlation-based transition models with the transport equation for the intermittency coefficient implemented in the commercial numerical code ANSYS Fluent ver. 17 were used for the comparison. The original \( \gamma \cdot \text{Re}_\theta \) transition model of Langtry and Menter [3] is based on transport equations for the intermittency coefficient \( \gamma \) and the transitional momentum Reynolds number \( \text{Re}_\theta \). The model is switched over to a simple algebraic transition model in the case of the transition in separated flow. The onset and the length of the transition region are controlled by threshold functions based on empirical correlations. The simplified \( \gamma \) transition model of Menter et al. [4] contains only one transport equation for the intermittency coefficient \( \gamma \). Both models are connected with the SST turbulence model.

### 3 Results

The above mentioned mathematical models were applied for the simulation of 2D compressible flow through the linear turbine blade cascade TR-Z-1. The scheme of the turbine blade cascade is shown in Fig. 1. Numerical results were compared with results of optical and pressure measurements, see Luxa et al. [19]. Predictions were carried out for the relative spacing \( t/c = 0.718 \), inlet angle \( \alpha_1 = 29.1^\circ \), the isentropic Reynolds number \( \text{Re}_{2\text{is}} = 1.66 \times 10^6 \) and for isentropic Mach number \( M_{2\text{is}} = 1.205 \).

The multi-block quadrilateral structured computational mesh refined near walls was used for numerical simulations by means of the in-house numerical code, the detail see in Fig. 2a. This approach allows the application of structured grids for complex geometries as well. The inlet of the computational domain was in the distance of 0.55\( c \) from the blade leading edge and the outlet in the distance 0.7\( c \) behind trailing edges.

The computational mesh used in the Fluent code was formed by quadrilateral elements and consists of 107808 cells. The grid elements are orthogonal and refined near walls, see Fig. 2b. The outlet part of the computation domain used in the Fluent code was extended up to 6\( c \) to reduce the effect of the shock-waves reflection from the domain outlet.

The inlet boundary conditions were defined by the constant total pressure, total temperature and inlet flow angle. The outlet boundary condition was defined by the constant static pressure determined according to the outlet isentropic Mach number. Periodicity conditions were applied on corresponding free side boundaries of the computation domain. The inlet free-stream turbulence parameters corresponding to turbulence intensity \( T_u = 1.5\% \) and the ratio of the turbulent and molecular viscosity \( \mu_t/\mu = 50 \) were chosen.

Numerical simulations were focused mainly on the modelling of the transition in the separated flow caused by the interaction of the shock wave with the laminar boundary layer. Realized simulations follow-up results obtained by Váchová et al. [20] by means of the \( \gamma \cdot \text{Re}_\theta \) transition model implemented in the commercial CFD code Numeca International version FINE/Turbo v9.1_3.
The flow field in the blade cascade is demonstrated in Figs. 3 and 4 by means of the interferometric and schlieren pictures, where the interaction of the inner branch of exit shock waves with the laminar boundary layer on the suction side can be clearly seen. It depends on the shock wave intensity if the separation with a short separated zone occurs.

The Mach number isolines and the numerical schlieren picture of the flow field obtained by the algebraic transition model with the Walker’s correlation for the transition length in the separation bubble are shown in Fig. 5. The same numerical results obtained by the $\gamma$ transition model of Menter et al. [4] are demonstrated in Fig. 6. It can be seen that the agreement of flow fields obtained by numerical simulations with experimental data is very good. All used transition models give very similar results. Small deviations can be found only in the zone of the shock wave interaction and following separation bubble due to various correlations for the length of separation-induced transition.
The distribution of the isentropic Mach number on the blade surface is shown in Fig. 7. The EARSM model connected with the algebraic transition model using the Walker’s correlation and the $\gamma$ transition model of Menter et al. [4] are compared with experimental data. The agreement of the simulation by used versions of transition models based on the algebraic and/or transport equation with experimental data is quite acceptable. Only small differences can be found in the zone of the shock-wave/boundary-layer interaction with the separation-induced transition.

The flow structure in the blade cascade apparent in the field of the Mach numbers and in the isentropic Mach number distribution is very well documented by means of the distribution of the skin friction coefficient. The skin friction coefficient is defined by the relation $C_f = \tau_w/p_{in}$ where $p_{in}$ is the inlet dynamic pressure. Results obtained by the algebraic transition model are shown in Fig. 8 for two various correlations of the transition length in the separation bubble. A very similar skin friction distribution was obtained by both modifications of the transition model with the transport equation for the intermittency coefficient.

All changes of the flow field structure appear in the skin friction distribution. The sudden change of the surface curvature of the blade suction side at the distance $x/b = 0.55$ results in the short laminar separation bubble and therefore in a change in the Mach number distribution. This tendency can be seen in the skin friction distribution in Fig. 7 obtained by both types of the transition model. This phenomenon was analysed in detail by Straka et al. [21].

A substantial impact on the flow in the blade cascade is caused by the shock-wave/boundary-layer interaction. The interaction of the inner branch of the exit shock wave with the laminar boundary layer on the suction side of the blade results in the flow separation and the subsequent reattachment. This phenomenon considerably influences the flow structure in the blade cascade and therefore the energy
losses as well. Further, a short separation region can be seen on the pressure side of the blade just upstream of the trailing edge.

The detail of skin friction distribution obtained by means of all tested versions of the transition models is shown in Fig. 9. The extent of the separation zone estimated from experimental results is marked by the rectangle. The extent of separation is approximately from $x/b = 0.71$ to 0.785. The all transition models give too short separation zone. Both transition models based on the transport equation for intermittency coefficient $\gamma$ give very similar results with a relative short separation bubble. The best agreement was achieved by the algebraic transition model with the Walker’s correlation for the transition length in the separation bubble as can be seen in Fig. 8a.

The existing empirical correlations for the separation-induced transition were derived mostly for the incompressible possibly subsonic flows around aerofoils where the separation and possible subsequent reattachment is caused by the adverse pressure gradient. The onset and the length of the transition is usually expressed by means of the boundary layer parameters in the separation position, see e. g. Mayle [16]. The separation zone induced by the shock wave/boundary layer interaction can be influenced by further parameters, at least by the Mach number. Therefore it is necessary to prove the existing correlations for transonic flows through blade cascades and to revise them.

Pressure measurements of the flow parameters behind the cascade were completed in the distance $0.4c$ behind the cascade outlet plane. The energy losses in the blade cascade were evaluated by means of the data reduction method proposed by Amecke and Šafařík [22]. The distribution of the total pressure behind the blade cascade is shown in Fig. 10. Experimental results are compared with numerical results for the algebraic transition model with the Walker’s correlation of the transition length in the separation bubble and for the $\gamma$ transition model of Menter et al. [4]. In addition to wake, the effect of the outer branch of the exit shock wave can be seen on the total pressure distribution obtained experimentally while both mathematical models give the wake behind the blade cascade only. Nevertheless, the agreement of
computed and measured energy losses is quite satisfactory. The best agreement of calculated and measured energy losses expressed by the ratio $\frac{\zeta}{\zeta_{exp}} = 0.987$ was achieved by the algebraic transition model with the correlation proposed by Walker [17] while the simplified $\gamma$ transition model of Menter et al. [4] gave $\frac{\zeta}{\zeta_{exp}} = 0.854$ only.

![Figure 10: Distribution of the total pressure behind the blade cascade](image)

**Conclusions**

The algebraic transition model with two various correlations for the transition length in the separation-induced transition was used to simulate transonic flow through the mid-section turbine blade cascade. Numerical results for the isentropic Mach number $M_{2is} = 1.205$ and for the isentropic Reynolds number $Re_{2is} = 1.66 \times 10^6$ were compared with two versions of the transition model with the transport equation for the intermittency factor. Numerical simulations were focused particularly on the effect of shock-wave/boundary-layer interaction on the flow structure especially on the interaction of the inner branch of the exit shock wave with the boundary layer on the blade suction side.

The predicted flow structure in the blade cascade corresponds well to experimental results. Simulations by means of several transitional models predicted the flow structure with two reflected shock waves and the expansion region between them. The crucial problem is the prediction of the separated region as all transition models predict rather shorter separation length. The best agreement was achieved by the algebraic transition model with the Walker’s correlations for the length of the separation bubble. The appropriate prediction of the separation zone is crucial for the estimation of the energy losses in the blade cascade.

The obtained results indicate that empirical correlations for the separation-induced transition due to the adverse pressure gradient should be revisited for the application for the simulation of transonic flows with the transition caused by the shock-wave/boundary-layer interaction. The separation zone induced by the shock wave/boundary layer interaction can be influenced by further parameters, at least by the Mach number. Therefore it is necessary to prove existing correlations for transonic flows through blade cascades and to revise them.

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**References**


