ON INFLUENCE OF VOCAL FOLD SIZE TO FREQUENCIES OF INDUCED VIBRATIONS

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Abstract

The paper deals with modelling of fluid-structure interaction (FSI). Particularly it is concerned with the example of human vocal folds and their vibration under excitation by the fluid flow. The vocal folds are described as an elastic body with the assumption of small displacements and therefore the linear elasticity theory is used. The viscous incompressible fluid flow is considered. For the purpose of the numerical solution the arbitrary Lagrangian-Eulerian method is used. The whole problem is solved by the finite element method based solver. The numerical results of the influence of vocal folds dimensions are presented.

Keywords: finite element method, 2D Navier-Stokes equations, vocal folds, aeroelasticity.

1 Introduction

The problem of fluid-structure interaction is very interesting complex problem. Its solution is important in many only technical applications, see [2]. Similar physical description of interaction between fluid flows and elastic bodies can be used in various studies dealing for example with airfoil or bridge stability as well as in investigation of blood flow in compliant arteries or in biomechanics of human voice production. This paper is concentrated on the biomechanics of human vocal folds, see e.g. [9]. Human voice is one of basic human being’s characteristics and it plays an important role in the quality of a human life. The air flow from lungs excites the vibrations of the vocal folds which produce the basic sound of human voice. The vibrations influence in reverse air flow and vice versa. Therefore we speak about coupled problem. The fluid-structure interaction (FSI) problem was studied in many papers, see e.g. [5], [7].

First, the interaction problem is described in the terms of linear elasticity theory and the incompressible viscous fluid flow modelled by the Navier-Stokes equations. Then the mathematical model is derived with the help of the arbitrary Lagrangian-Eulerian (ALE) method which enables to handle with the time-dependent domain. The numerical algorithm is based on the finite element method (FEM). For the fluid solver the P1-bubble/P1 elements and for the elastic part piecewise linear P1 elements are implemented. For the coupled problem the fully implicit scheme is used. The attention of this paper is focused on investigation of influence of different vocal fold sizes and associated eigenfrequencies.

2 Mathematical model

For the sake of simplicity we restrict ourselves to the two-dimensional model. Scheme of the model domain is shown in Fig. 1. Quantities connected with fluid and structure are labelled by indexes $^f$ and $^s$, respectively. The boundary of fluid domain $\partial\Omega^f_t$ at arbitrary time $t$ consists of the inlet part $\Gamma^f_{\text{In}}$, the outlet part $\Gamma^f_{\text{Out}}$, and $\Gamma^f_{\text{Dir}} \cup \Gamma^f_{\text{W}}$. The solid part $\Gamma^s_{\text{Dir}}$ represents fixed walls of the glottal channel and the only time dependent boundary $\Gamma^s_{\text{W}}$ represents the interface on the vibrating vocal folds. The undeformed (reference) shape of the domain is marked by index $\text{ref}$.

The Lagrange coordinates are used for solution of the structure motion. The fluid flow is described in Eulerian coordinates and later solved using the arbitrary Lagrangian Eulerian coordinates, see [8].
2.1 Elastic body.

Vocal folds are modelled as an elastic body. It’s deformation is given by, see e.g. [1]

\[ \rho^s \frac{\partial^2 \mathbf{u}}{\partial t^2} - \text{div}_X \tau^s = \mathbf{f}^s \text{ in } \Omega^s \times (0,T), \]  

where \( \rho^s \) is the structure density, the tensor \( \tau^s \) is the Cauchy stress tensor with the components \( \tau^s_{ij} \), the vector \( \mathbf{f}^s \) is the volume density of an acting force and \( X = (X_1, X_2) \) are the reference coordinates \( X \in \Omega^s_{\text{ref}} \). Using the generalized Hook law for an isotropic body, the components of the stress tensor \( \tau^s_{ij} \) can be expressed as

\[ \tau^s_{ij}(\mathbf{u}) = \lambda^s (\text{div} \mathbf{u}) \delta_{ij} + 2 \mu^s e^s_{ij}, \]  

where \( e^s_{jk} \) is the strain tensor for small displacements \( e^s_{jk} = \frac{1}{2} \left( \frac{\partial u_j}{\partial X_k} + \frac{\partial u_k}{\partial X_j} \right) \) and \( \lambda^s, \mu^s \) are Lame coefficients. They depend on Young modulus of elasticity \( E^s \) and the Poisson’s ratio \( \sigma^s \), see [1].

Eq. 1 is supplied with the following initial and boundary conditions

\begin{align*}
    a) & \quad \mathbf{u}(X,0) = \mathbf{u}_0(X) \quad \text{for } X \in \Omega^s, \\
    b) & \quad \frac{\partial \mathbf{u}}{\partial t}(X,0) = \mathbf{u}_1(X) \quad \text{for } X \in \Omega^s, \\
    c) & \quad \mathbf{u}(X,t) = \mathbf{u}_{\text{Dir}}(X,t) \quad \text{for } X \in \Gamma^s_{\text{Dir}}, \quad t \in (0,T), \\
    d) & \quad \tau^s_{ij}(X,t) n^s_j(X) = q^s_i(X,t) \quad \text{for } X \in \Gamma^s_{\text{W}}, \quad t \in (0,T),
\end{align*}

where the boundaries \( \Gamma^s_{\text{W}}, \Gamma^s_{\text{Dir}} \) are shown in Fig. 1, \( \overrightarrow{n}^s(X) = (n^s_1, n^s_2) \) is the unit outer normal to \( \partial \Omega^s_{\text{ref}} \) and \( q^s(X,t) = (q^s_1, q^s_2) \) is the aerodynamic force described later.

2.2 ALE method.

We use ALE method to trace changes of fluid domain in the time variable region, see e.g. [8]. We suppose that there exists a diffeomorphism \( A_t \) which maps the reference (undistorted) domain \( \Omega^f_{\text{ref}} \) to time-dependent domain \( \Omega^f_t \) at any instant time \( t \in (0,T) \). In addition we assume that the mapping \( A_t \) satisfy

\[ \frac{\partial A_t}{\partial t} \in C(\Omega^f_{\text{ref}}), \quad A_t(\partial \Omega^f_{\text{ref}}) = \partial \Omega^f_t, \quad t \in (0,T). \]  

Further, the domain velocity \( \mathbf{w}_D \) is defined by

\[ \mathbf{w}_D(x,t) = \frac{\partial}{\partial t} A_t(X), \quad t \in (0,T), \quad x = A_t(X) \in \Omega^f_t. \]  

Afterwards we introduce ALE derivative as the time derivative with respect to a fixed point \( X \in \Omega^f_{\text{ref}} \). The ALE derivative satisfies

\[ \frac{D^A}{Dt} f(x,t) = \frac{\partial f(A_t(X),t)}{\partial t} = \frac{\partial f}{\partial t}(x,t) + \mathbf{w}_D(x,t) \cdot \nabla f(x,t), \]  

where \( f \) is
2.3 Fluid flow.

The fluid flow in the time dependent domain \( \Omega_f^t \) is described by Navier-Stokes equations, see e.g. [7]

\[
\frac{D^4 \mathbf{v}}{D t^4} + ((\mathbf{v} - \mathbf{w}_D) \cdot \nabla) \mathbf{v} - \nu^f \Delta \mathbf{v} + \nabla p = \mathbf{g}^f, \quad \text{div } \mathbf{v} = 0 \quad \text{in } \Omega_f^t.
\]

where \( \mathbf{v} \) is the fluid velocity, \( \nu^f \) is the kinematic viscosity, \( p \) denotes the kinematic pressure and \( \mathbf{g}^f \) are volume forces. The formulation of the problem is completed by the initial and boundary conditions

\[
a) \quad \mathbf{v}(x, t) = \mathbf{w}_D(x, t) \quad \text{for } x \in \Gamma_{\text{Dir}}^t \cup \Gamma_{\text{Wt}}, \ t \in (0, T), \\
b) \quad \mathbf{v}(x, t) = \mathbf{v}_{\text{Dir}}(x, t) \quad \text{for } x \in \Gamma_{\text{Int}}^t, \ t \in (0, T), \\
c) \quad (p(x, t) - p_{\text{ref}}) \mathbf{n}^f - \nu^f \frac{\partial \mathbf{v}}{\partial \mathbf{n}^f}(x, t) = 0, \quad \text{for } x \in \Gamma_{\text{Out}}^f, \ t \in (0, T), \\
d) \quad \mathbf{v}(x, 0) = \mathbf{v}_0(x) \quad \text{for } x \in \Omega_f^0,
\]

where \( \mathbf{n}^f \) denotes outer unit normal to \( \partial \Omega_f^t \) and \( p_{\text{ref}} \) is a reference pressure.

2.4 Coupling conditions.

As the fluid-structure interaction is a coupled problem, it means that solutions in the fluid and the structure domain depend on each other. They are connected via boundary conditions. Moreover, the location of the interface \( \Gamma_{\text{Wt}} \) at time \( t \) is not known and it depends on establishing force equilibrium between aerodynamic and elastic forces. On the other hand these forces are also dependent on the position and the velocity of the interface.

The location of the interface \( \Gamma_{\text{Wt}} \) at time \( t \) is given by the deformation \( \mathbf{u} \), so

\[
\Gamma_{\text{Wt}} = \{ x \in \mathbb{R}^2 | x = X + \mathbf{u}(X, t), \ X \in \Gamma_{\text{Wt}} \ref \}. \quad (9)
\]

The deformation \( \mathbf{u} \) is influenced by the aerodynamic force \( \mathbf{q}^* \) acting on the structure interface

\[
q_i^*(X, t) = - \sum_{j=1}^2 \sigma_{ij}^f(x) n_j^f(x), \quad i = 1, 2, \quad X \in \Gamma_{\text{Wt}} \ref, \quad x = X + \mathbf{u}(X, t), \quad (10)
\]

where \( \sigma_{ij}^f = -\rho^f \mathbf{p} + \rho^f \nu^f (\frac{\partial \mathbf{u}}{\partial x_j} + \frac{\partial \mathbf{u}_j}{\partial x_i}) \) are components of the fluid stress tensor and \( \mathbf{n}^f(x) = (n_1^f, n_2^f) \) denotes the components of the unit normal (here to the interface \( \Gamma_{\text{Wt}} \)) oriented out of \( \Omega_f^t \).

The boundary condition for fluid on the interface \( \Gamma_{\text{Wt}} \), or solid wall \( \Gamma_{\text{Dir}}^t \) is prescribed to be in correspondence with motion of the elastic body, e.g. it is equal to the velocity of the structure domain or zero as it was done in Eq. 8 a).

3 Numerical model

First, Eq. 1 is discretized in time and in space by the FEM. Similarly, the time discretized Eq. 7 is approximated in space by FEM. Both models are solved with the equidistant time steps \( \Delta t > 0 \).

3.1 Elastic structure.

For the application of FEM the weak formulation of Eq. 1 is introduced. The solution \( \mathbf{u} \) is now sought in the space \( \mathbf{V} = V \times V \), where \( V = \{ f \in W^{1,2}(\Omega^s) | f = 0 \text{ on } \Gamma_{\text{Dir}} \} \), and \( W^{k,p}(\Omega) \) denotes the Sobolev space. Eq. 1 with the aid of Hook law Eq. 2 is reformulated in the weak sense as (see e.g. [10])

\[
\int_{\Omega^s} \rho^s \frac{\partial^2 \mathbf{u}}{\partial t^2} \cdot \mathbf{w} \, dx + \int_{\Omega^s} (\lambda^s (\text{div } \mathbf{u}) \delta_{ij} + 2\mu^s \varepsilon_{ij}^s(\mathbf{u})) \varepsilon_{ij}^s(\mathbf{w}) \, dx = \int_{\Gamma_{\text{Neu}}} \mathbf{r}^s \cdot \mathbf{w} \, dS. \quad (11)
\]
Furthermore, the space $\mathbf{V}$ is approximated by the space $\mathbf{V}_h \subset \mathbf{V}$ with $\dim \mathbf{V}_h = N_h$. Thus the discrete solution can be written as $u_h(x,t) = \sum_{j=1}^{N_h} a_j(t) \Phi_j(x)$, where $\Phi_j$ denotes the standard FE base functions. Using this expression and labeling $q$ the weak formulation of Eq. 15 is acquired by the multiplication of the first Eq. 15 by $\int_{\Omega^f} \lambda^s \Phi_j \delta t_1 + 2 \mu^s e^s_{ii}(\Phi_j) e^s_{ii}(\Phi_i) \, \mathbf{d}x$, where $\lambda^s$ is the shear modulus of the material, $\mu^s$ is the shear viscosity, $e^s_{ii}$ is the shear strain, and $\delta t_1$ is the time increment. The approximative solution $u$ can be written as

$$ M \ddot{u} + K \dot{u} = \mathbf{b}(t), $$

where the elements of the matrices $M = (m_{ij})$, $K = (k_{ij})$ and vector $\mathbf{b}(t) = (b_i)$ are given by

$$ m_{ij} = \int_{\Omega^s} \mu^s \Phi_j \cdot \Phi_i \, \mathbf{d}x, \quad k_{ij} = \int_{\Omega^s} (\lambda^s \text{div} \Phi_j \delta_1 + 2 \mu^s e^s_{ii}(\Phi_j) e^s_{ii}(\Phi_i) \, \mathbf{d}x, \quad b_i = \int_{\Omega^s} \mathbf{f} \cdot \Phi_i \, \mathbf{d}x + \int_{\Gamma_{\text{hac}}} q \cdot \Phi_i \, \mathbf{d}s. $$

In the practical computation the Lagrange finite elements of the first order are used, which give the first order of accuracy in space. The ordinary differential Eqs. 12 of the second order are then solved by Newmark method, for details see [10].

### 3.2 Flow model.

For the flow model we start with discretization of Eq. 7 in time. For time discretization the backward differentiation formula of second order (BDF2) is used, see [5]. So the ALE derivative is approximated as

$$ \frac{D^A}{Dt} (x_{n+1}, t_{n+1}) \approx \frac{3 \mathbf{v}^{n+1}(x_{n+1}) - 4 \mathbf{v}^n(x_n) + \mathbf{v}^{n-1}(x_{n-1})}{2 \Delta t}, $$

where for a fixed time instant $t_{n+1}$ we denote $\mathbf{v}(x_{n+1}) = \mathbf{v}^i(x_i)$ for $x_i = A_i \left( A_{t_{n+1}}^{-1}(x_{n+1}) \right), i = n-1, n$, and $x_{n+1} \in \Omega_{t_{n+1}}^f$. In further text we will omit the time index $n+1$, e.g. $\Omega^f := \Omega_{t_{n+1}}^f$. After application of the scheme in Eq. 7 we get

$$ \frac{3 \mathbf{v} - 4 \mathbf{v}^n + \mathbf{v}^{n-1}}{2 \Delta t} + ((\mathbf{v} - \mathbf{w}_D) \cdot \nabla) \mathbf{v} - \nu^f \Delta \mathbf{v} + \nabla \mathbf{p} = \mathbf{g}^f, \quad \nabla \cdot \mathbf{v} = 0. $$

**Weak formulation.** As before the velocity solution $\mathbf{v}$ at time $t_{n+1}$ is sought in the space $W^{1,2}(\Omega_{k+1}^f)$ and the pressure $\mathbf{p}$ in the space $\mathbf{M} = L^2(\Omega_{k+1}^f)$. Further, the test function space $\mathbf{X}$ is defined by

$$ \mathbf{X} = \mathbf{X} \times \mathbf{X}, \quad \mathbf{X} = \{ f \in W^{1,2}(\Omega_{k+1}^f) \mid f = 0 \text{ on } \Gamma_{\text{Dir}} \cup \Gamma_{\text{In}} \cup \Gamma_{\text{Out}}^f \} \subset W^{1,2}(\Omega_{k+1}^f). $$

The weak formulation of Eq. 15 is acquired by the multiplication of the first Eq. 15 by $\Phi \in \mathbf{X}$, second by $q \in \mathbf{M}$, addition, integration over the whole domain $\Omega^f$ and by using the Green’s theorem. This leads to the following equation

$$ \left( \frac{3 \mathbf{v} - 4 \mathbf{v}^n + \mathbf{v}^{n-1}}{2 \Delta t}, \phi \right)_{\Omega^f} + ((\mathbf{v} - \mathbf{w}_D) \cdot \nabla) \mathbf{v}, \phi \right)_{\Omega^f} + \nu^f \left( \nabla \mathbf{v}, \nabla \phi \right)_{\Omega^f} - \left( p, \text{div} \phi \right)_{\Omega^f} + (q, \text{div} \mathbf{v})_{\Omega^f} = (\mathbf{f}, \phi)_{\Omega^f} - (\mathbf{p}_{\text{ref}}, \phi \cdot \nabla \phi)_{L^2(\Gamma_{\text{out}}^f)}, $$

which should be satisfied for any $\phi \in \mathbf{X}$ and $q \in \mathbf{M}$, where $\langle \cdot , \cdot \rangle_{\Omega^f}$ stands for the scalar product in $L^2(\Omega^f)$ or $[L^2(\Omega^f)]^2$ spaces.

The FEM approximates the spaces $\mathbf{X}$ and $\mathbf{M}$ by the finite dimensional spaces $\mathbf{X}_h$ and $\mathbf{M}_h$. One of the aspects of the FEM is that the choice of spaces $\mathbf{X}_h$, $\mathbf{M}_h$ must satisfy the Babuška-Brezzi condition, see [4]. In this article P1-bubble/P1 elements (see Fig. 2) are used, which according to [4] satisfy this condition. The approximative solution $\mathbf{v} \approx \mathbf{v}_h, \mathbf{p} \approx p_h$ of Eq. 17 can be expressed as a linear combination of vectors of coefficients $\beta = (\beta_j), \gamma = (\gamma_j)$ and basis functions $\phi_j \in \mathbf{X}_h$, $q_j \in \mathbf{M}_h$. Then the substitution of linear combinations into Eq. 17 gives the system of non-linear equations

$$ \begin{pmatrix} A(\beta) & B \end{pmatrix} \begin{pmatrix} \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} g \\ 0 \end{pmatrix}, $$

where

$$ A(\beta) = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^T & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \cdots & \mathbf{b}_n \end{pmatrix}, \quad \mathbf{b}_i = \begin{pmatrix} b_{i1} \\ b_{i2} \\ \vdots \\ b_{in} \end{pmatrix}, \quad g = \begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{pmatrix}. $$
where $A(\beta) = \frac{1}{\Delta t} M + C(\beta) + D$. The elements of the matrices $M = (m_{ij}), C = (c_{ij}), D = (d_{ij}), B = (b_{ij})$ and components of vector $g = (g_i)$ are given by

$$m_{ij} = \frac{3}{2} (\varphi_j, \varphi_i)_{\Omega f}, \quad c_{ij} = ((\mathbf{v}_h - \mathbf{w}_D) \cdot \nabla) \varphi_j, \varphi_i)_{\Omega f}, \quad d_{ij} = \nu (\nabla \varphi_j, \nabla \varphi_i)_{\Omega f}, \quad (19)$$

$$b_{ij} = (\mathbf{g}_h, \nabla \varphi_i)_{\Omega f}, \quad g_i = (\mathbf{g}_f, \varphi_i)_{\Omega f} - (p_{ref}, \varphi_i)_{L^2(\Gamma_{out})} + \left( \frac{4 |\nabla u^n| - |\nabla u^{n-1}|}{2 \Delta t}, \varphi_i \right)_{\Omega f},$$

and $\mathbf{v}_h = \sum_k \beta_k \varphi_k$. For the solution of the non-linear system 18 the Oseen linearizations are used. The solution of the system of linear equations is performed by mathematical library UMFPACK, see [3].

### 3.3 Coupled problem.

Last part of our algorithm is the coupling both solvers via aerodynamic forces. They are evaluated from the known fluid velocity and pressure values at adjacent triangles to interface $\Gamma_W$ according to Eq. 10. The values of aerodynamic forces are computed in vertices on the interface with the help of numerical quadrature and these values are provided to elastic solver as a discrete version of the Neumann boundary condition (3 d). The algorithm solving coupled problem is implemented in the fully implicit form, see [10].

### 4 Numerical simulations

For the numerical tests the vocal fold model M5 suggested by paper [6] was used. The computational domain scheme together with a triangulation is shown in Fig. 3. The investigation of dimensions influence was done as follows. The computations were performed on three meshes of the same geometry with varying dimensions of the vocal folds. In the case A) the dimensions were chosen as it is shown in Fig. 3, i.e. the height of vocal fold equals 11 mm. In the case B) it is 8 mm and in the case C) 6 mm. The M5 model was used. The proportions of the fluid domain were treated similarly except the height of glottis, i.e. the narrowest part of channel was kept constantly

![Figure 3](image-url)
3 mm high in all uses. This is motivated by article [11], where it is concluded that normal vocal folds are $6 - 8$ mm high. On the other hand all of our previous results, for example see [10], were performed with height 11 mm as it is suggested in [6].

In this first approach only one half of the channel was used and the symmetry of the solution was prescribed. First, the structure model was numerically approximated. Next the solution of the coupled problem was performed.

### 4.1 Modal analysis.

The eigenfrequencies and eigenmodes of the vocal fold model were determined by modal analysis. The solution of the system of ordinary differential equations $\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = 0$, where the matrices $\mathbf{M}$ and $\mathbf{K}$ are given by (13), is sought in the form $\mathbf{u}(t) = e^{i\omega t}\mathbf{U}$, where $\mathbf{U}$ is a vector with $N_h$ components. It leads to a generalized eigenvalue problem $(\mathbf{K} - \omega^2\mathbf{M})\mathbf{U} = 0$.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{1}^{MA}$</td>
<td>57.39</td>
<td>78.92</td>
<td>105.22</td>
</tr>
<tr>
<td>$f_{2}^{MA}$</td>
<td>116.93</td>
<td>160.77</td>
<td>214.37</td>
</tr>
<tr>
<td>$f_{3}^{MA}$</td>
<td>124.64</td>
<td>171.37</td>
<td>228.50</td>
</tr>
<tr>
<td>$f_{4}^{MA}$</td>
<td>199.02</td>
<td>273.66</td>
<td>364.84</td>
</tr>
</tbody>
</table>

Table 1: The five lowest eigenfrequencies obtained from the modal analysis of model M5 for different height of vocal folds.

The first four resulting eigenmodes and their frequencies for case A) are shown in Fig. 4. These results were obtained for the Young’s modulus of elasticity $E^s = 100$ kPa, $\sigma^s = 0.4$ in a thin layer along the interface $\Gamma_{W-ref}$ (epithelium - shown in red colour) and $E^s = 12$ kPa, $\sigma^s = 0.4$ otherwise inside $\Omega_{s-ref} -$ muscle (green).

It can be seen that the first mode represents mainly horizontal motion while the second represents mainly the vertical motion. The other eigenmodes are more complicated, see Fig. 4. It corresponds well with results shown in [12], where the first two eigenmodes also correspond to the horizontal and vertical motions.
The results of other two cases are summarized in Table 1. As theory predicts the eigenfrequencies depend on dimensions linearly, which confirms our results.

4.2 FSI test.

The interaction between the fluid and the elastic body was tested in the case of prescribed zero boundary and initial conditions for the structure. The fluid setting was following: The kinematic viscosity was set $\nu_f = 1.5 \times 10^{-5}$ m/s$^2$ and the inlet boundary condition had a half of parabolic profile in $x$-component with the maximum 1.5 m/s. The interaction was enabled immediately after the start of the simulation. In every time step ($\Delta t = 2 \times 10^{-4}$ s) the convergence of interface forces took approximately 5 inner iteration steps.

It can be seen that the flow excited a periodic vibration of the vocal fold with small amplitudes for all considered cases. The example of the time signal of chosen point on the top of vocal fold in case C) is shown in Figs. 5 and 6.

The time signal of a chosen point was analyzed by Fourier transformation, see Fig. 5 and 6. The obtained dominant frequencies for the motion of this point are shown in Table 2. The frequencies agree in given accuracy with frequencies obtained by the modal analysis summarized in Table 1 on previous page. This also corresponds well with the results of [12], where also primarily the first two eigenmodes were excited during FSI.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure5.png}
\caption{Time evolution of $x$-displacement of point for the FSI test (left) and it’s Fourier transformation with dominant frequency $f = 103.9$ Hz (right).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure6.png}
\caption{Time evolution of $y$-displacement of point for the FSI test (left) and it’s Fourier transformation with dominant frequencies (right).}
\end{figure}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
& $x$-direction & & \\
& A & B & C \\
\hline
$f_1$ & 56.86 & 79.0 & 104.6 \\
$f_2$ & 123.5 & 160.0 & 212.95 \\
$f_3$ & 238.2 & 170.0 & 226.848 \\
$f_4$ & 288.2 & 325.0 & 419.4 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
& $y$-direction & & \\
& A & B & C \\
\hline
$f_1$ & 56.86 & 79.0 & 104.6 \\
$f_2$ & 115.7 & 160.0 & 213.0 \\
$f_3$ & 198.0 & 271.0 & 358.2 \\
$f_4$ & 265.7 & 363.0 & 477.8 \\
\hline
\end{tabular}
\end{table}

Table 2: The dominant eigenfrequencies obtained by the spectral analysis of point motion in FSI test for different simulation cases. The left results are obtained from $x$-component of time signal, where $y$-component are processed on right.
5 Conclusion

The paper presented the formulation of the FSI problem focused on the example of human fold vibration in airflow. The formulation is based on FEM and the ALE method and the developed numerical scheme was implemented in an own program. The special attention was devoted to comparison of results for different vocal folds sizes. The frequencies obtained with modal analysis were linear proportional to dimensions of vocal fold. The same results were confirmed later by FSI simulation, where the frequency of chosen point was investigated. Therefore it can be concluded that dimensions of vocal folds significantly alter the dominant eigenfrequencies.

For the future it remains the question how much appropriate is our physical model, which is based on linear elasticity model and Navier-Stokes equations without considering turbulence and if the laboratory experiments would lead to same change of eigenfrequencies. On the other hand all key part of the basic behaviour of FSI is satisfactory represented in our model although with linear accuracy.

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