FLOW PATTERNS IN A COMBINED TAYLOR–COUETTE GEOMETRY

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Abstract

The Taylor Couette problem, which is the fluid motion in an annulus between two concentric rotating bodies, is a convenient flow system to study the laminar-turbulent transition and has a fundamental interest in the wavenumber selection processes. This paper presents a numerical study of conical Taylor-Couette flows when the inner cylinder is a regular straight cylinder, but the outer cylinder has a tilted, conical shape. The apex angle between the inner cylinder and the outer cone is varied between 0 and 12 degrees. The parameter that determines the flow regimes is the Taylor number based on the angular velocity of the inner cylinder. The calculations are carried out using a three-dimensional CFD of incompressible viscous flow. Computations for the onset of Taylor vortices in the classical configuration with straight cylinders show good agreement with experimental data. For the case of a conical outer cylinder, calculations show a decrease in the critical Taylor number for the onset of the first instability along with the number of rolls with the apex angle. The main result of this geometrical modification is that the gap width varies in height, and the Taylor vortices then vary in size, being large where the gap is wide, and small where the gap is narrow. Pressure distribution is also computed.

Keywords: CFD simulations, Flow patterns, Wave number selection, Combined Taylor-Couette flow system, Apex angle, Instability

1 Introduction

The stability and transitions of viscous incompressible flow in an annulus between two coaxial rotating bodies, termed Taylor-Couette flow (TCF), presents a paradigm for experimental, theoretical and numerical studies of the hydrodynamic stability and the transition to turbulence. In addition, Taylor-Couette flow system is typically closed environment, where the working fluid is confined radially between the cylinders, and axially by endplates. This flow is a natural choice as a system for study because its high symmetry has made the study of the laminar regimes highly successful compared to more general flows. Furthermore, the Taylor Couette system has tremendous interests for various applications. Typical examples are filtration, tribology, wastewater treatment, cell-cultivation, biomedical, mixing processes, photo-catalysis, liquid-liquid extraction and rheology.

So far, an abundance of theoretical, experimental and numerical studies on the formation of Taylor vortices between two rotating cylinders, circular Couette flow, with either or both of them rotating have been presented with various geometric parameters, i.e., aspect ratio and radii ratio, focusing on the flow behavior and the transition to turbulence under different boundary conditions [1-18]. The researchers have concluded that the various flow regimes occurring in the cylindrical Taylor-Couette system depend on the radii ratio and aspect ratio of this geometry.

Because the majority of the pioneering works were focused on the flow patterns between two coaxial rotating cylinders, later researches are extended to other flow configurations where the Taylor vortices may be appeared. Modifying circular Couette flow by replacing the straight cylinders by cones, known as conical Taylor Couette flow (CTCF), deeply affects the flow patterns and the appearance of different instabilities. Differing from circular Couette flow (CCF), the radii in CTCF vary axially and the basic flow is fully three-dimensional which may therefore produce more complex flow structures.

Several studies available in the literature have been focused on this type of flow treating the influences of free surface, gap width, cone angle, cone height, and rate of acceleration on the flow patterns [19-31].

Note that in CCF and CTCF the gap width remains constant. If the gap width is no longer constant such as the flow in the annulus between combinations of circular and conical cylinders, a rich variety of
interesting flow modes can be occurs. So far, the flow behavior and the selection of wavenumber in a variety of inner or outer noncircular cylinder arrangements have been the subject of numerous studies [32-41]. Furthermore, Cannel et al. [42] and have presented experimental results on the wavenumber selection in a Taylor–Couette system where a section of the gap was tapered. They found that a sufficiently slow ramp connecting the supercritical region to the subcritical region resulted in the selection of a unique wavelength. They also shown that even very small opening angle of the outer cylinder wall could lead to a multiplicity of state selections. In addition, Noui-Mehidi et al. [43] presented numerical results of the effect of miss-alignment of the outer cylinder wall on the flow instability in a short circular Couette geometry. This study revealed that the bifurcation from a particular mode to another one occurs at a range of specific values of the opening angle of the outer cylinder. Sprague et al [44, 45] have investigated, both numerically and experimentally, the flow behavior when only the geometry of the inner cylinder is modified or else the inner and outer radii varied axially. They have shown that the onset of Taylor vortices of different wavelength can be produce simultaneously.

Recently, Raju [46] shown that if one of the cylinders has a step change in radius, there can be even or odd number of vortices in the wide and narrow gap regions.

Despite the numerous investigations dealing with the laminar-turbulent flow transition between two concentric rotating cylinders or cones, the flow between combinations of circular and conical cylinders remain still widely unexplored and there are only a few studies on numerical modeling devoted to this issue. The motivation of this paper, in addition to its connection with the classical Taylor-Couette problem, is to investigate numerically the fluid motion in an annulus between cylinder-cone combinations by using CFD simulations for a three dimensional viscous and incompressible flow. To our knowledge, this type of investigation has not yet been considered and it is important to make this approach to assess the impact of the geometrical modification on the existence and nature of Taylor vortices as well as the wavenumber selection processes.

2 Physical model and mathematical formulation

2.1 Physical model

Consider a viscous incompressible flow between cylinder–cone combinations. The inner cylinder rotates with a constant angular velocity \( \Omega_1 \) around the vertical z-axis, from west to east. The outer conical cylinder, bottom and the upper surfaces are stationary, as illustrated in Figure. 1. The opening angles of the outer cylinder are increased stepwise, for which the configuration changed from two rotating coaxial circular cylinders toward a rotating cylinder in a stationary cone. The angle of conic ranging from 0 up to 12°, resulting in a gap width not perfectly parallel.

In this framework, the numerical results are described by the following non-dimensional parameters:

- Average gap width: \( d = L_2 - L_1 \);
- Aspect ratio: \( \Gamma = H/d \);
- Radius ratio: \( \eta = L_1 / L_2 \);
- Ratio of the gap to the radius of the inner cylinder: \( \delta = d/L_1 \);
- Apex angle: \( 0 \leq \alpha \leq 12^\circ \);
- Taylor number: \( T_a = \frac{\Omega_1 L_1 d}{v} \sqrt{\frac{d}{L_1}} \)

Where \( L_1 = R_1 \) is the radius of the inner cylinder, \( L_2 = (R_{2\text{min}} + R_{2\text{max}})/2 \) is the average radius of the outer cone with, \( R_{2\text{min}} = R_1 + d_0 \) denote the radius of outer cylinder for \( \alpha = 0 \) and \( R_{2\text{max}} = R_{2\text{min}} + H \tan \alpha \) is the radius of cone when \( \alpha \neq 0 \). \( H \) is the height of cylinder, \( \Omega_1 \) is the angular velocity of the inner cylinder and \( v \) is the kinematic viscosity.
We note that the annular gap increases from the upper base downward when the apex angle increases slowly. Therefore, the definition of the Taylor number for all cases studied above is related to the gap defined at the upper base, which is the same for all configurations.

2.2 Mathematical formulation

In this numerical investigation, the flow field is presented in an axisymmetric concentric annulus configuration. The basic governing equations of the fluid dynamics are based on the fundamental physical laws of conservations. The equations of the continuity and momentum for an incompressible viscous flow are written as follow:

\[ \nabla \cdot \mathbf{u} = 0 \]  
\[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} \]  

Let \((u_r, u_\theta, u_z)\) the physical components of the velocity \(\mathbf{u}\) in cylindrical coordinates \((r, \theta, z)\). \(P, \rho,\) and \(\nu\) denote pressure, density, and kinematic viscosity of the fluid, respectively. The boundary conditions used in this calculation are given as follow:

- For the cylinders:
  \( u_\theta = R_1 \Omega_1 \) and \( u_r = u_z = 0 \) if \( r = R_1 \);
  \( u_r = u_\theta = u_z = 0 \) if \( r = R_2 \);

- For the bottom and upper surfaces:
  \( U = V = W = 0 \) for \( Z=0 \) and \( Z=H \)

3 CFD modeling

3.1 Mesh Generation

The 3D geometries are meshed with hexahedral cells (structured mesh). The computational domain is divided into a number of grids in the radial \((r)\), azimuthal \((\theta)\) and axial \((z)\) directions, respectively. The mesh cells were varied from coarse to fine size. The mesh size is uniform in the axial and azimuthal directions, but it is stretched near both walls in the radial direction where gradients are steep, as shown in fig.2. For the calculated results, the cells number determined by grid-refinement study ranged from 160 to 240 in the axial direction, from 20 to 36 in the radial direction, and from 180 to 260 in the azimuthal direction for different numerical results presented below. The outer element, the upper and the bottom plates are modeled as fixed walls with no-slip boundary conditions; while the moving wall boundary condition is applied for inner cylinder with no-slip condition is imposed. The apex angle of the outer conical cylinder is varied from \(0\) up to \(12^\circ\) for different set of simulation run. The angular velocity of the inner cylinder is increased quasi-steadily, from rest up to the occurrence of the Taylor vortices.
3.3 Method of Solution
The Navier-Stokes equations have been solved using the CFD code based on the finite volume formulation. The discretization scheme chosen for the pressure is the "second order" model and the momentum equations have been discretized with third order MUSCL scheme. A PISO algorithm, Pressure-Implicit with Splitting of Operators, is used to link pressure and velocity. The convergence criteria used to terminate the simulations was $10^{-6}$ for continuity and three velocity components.

3.4 Validation
The model validation has been presented in terms of comparison of the onset of first instability, TVF, with experimental data of Fenstermacher et al [47]. The critical Taylor number computed for this study is 41.6, which is close to the value of 41.2 reported by Fenstermacher et al [47] for the same geometrical parameters. Figure 3 shows very good agreement between the CFD predictions and the experimental data of Fenstermacher et al [47] within 1%.

4. Main numerical results
4.1 Flow patterns
The flow patterns obtained for different apex angles are reported in figures 4 and 5 representing the contours of static pressure and streamlines in the outer cone, respectively. For the classical case, $\alpha=0$, time-independent Taylor vortex flow appears according to a clearly identified scenario when the Taylor number reaches a critical value, as reported previously in the literature.

However, the number of rolls characterizing the onset of the first instability decreases drastically when the apex angle increases, as can be seen in figure 4. We note also that when the $\alpha$ increases, the roll near the upper base is less intense compared to other rolls which is due to the variable gap width. The centrifugal force is more important in the lower region that in the upper zone. The streamlines contours depict the Taylor vortex pattern as alternate cells, where consecutive cells move in the same direction at their meeting point, as shown in figure 5. The flow is circumferential due to the shearing action of the rotating inner cylinder.
4.2 Taylor vortices behavior

Figure 6 shows the Taylor vortices structures represented by the contours of $z$ vorticity in $(r, z)$ plane for several $\alpha$. At $\alpha = 0$, when the Taylor number reaches the critical value, a series of periodic vortices are
developed between the concentric cylinders. The flow pattern consists of pairs of nearly circular cells and resembles counter-rotating toroids stacked in the axial direction, known as Taylor vortices. For this case, there are 20 cells (10 wave) of equal size and intensity inside the annulus. Thus, the constant gap guarantees the same centrifugal forces at every point in the z-direction.

For $\alpha \neq 0$, it is observed that, when the apex angle increases, the number of wave decreases from 10 up to 4 at $\alpha = 12^\circ$. The number of vortices filling the entire gap is inversely proportional to the apex angle. For $\alpha = 0$ (CCF), 20 vortices are established, whereas 18, 16, 14, 12, 10 and 8 vortices are observed for $\alpha = 20'$, 2, 4, 6, 8, 10° and 12° respectively, as can be seen figure 7 showing the variation of wave number $n$ versus apex angles.

Furthermore, the Taylor vortices are deformed, having larger differences in their axial extension. The size of the cells is not uniform over the whole axial length and varies from one angle to another. The variation of the vortices size may due to the non-constant annular gap, i.e. the centrifugal force is stronger in the larger than in the smaller part of the gap. The size of the bottom vortex is greater than the upper vortex. The effect of the growing bottom vortex could be seen clearer in the case of higher values of $\alpha$. Because the system is closed and the vortices are propagating, vortices periodically form in the region of large radius and merge with endwall vortices in the region of small radius.

It can be concluded from these figures that the existence and wavelength of Taylor vortices depends of the opening angle of the outer cylinder. This result could provide useful guidance for the design and optimization of semi-batch Taylor–Couette devices.

We note also that the transition mechanism, occurrence of Taylor vortices, for the case of $\alpha \neq 0$ is not similar to classical case, $\alpha = 0$, where the cylinder walls are not perfectly parallel. In this case, $\alpha \neq 0$, first vortex appears at the location of the largest gap size, and regions with and without vortices could be observed within the gap. Finally, at critical Taylor numbers, the whole annulus is filled with vortices.

![Figure 6. Taylor vortices for different apex angle / Z vorticity profile in (r, z) plane.](image-url)
4.3 Pressure distributions

Figure 8 shows the axial distribution of the mean pressure plotted in dimensionless form at different apex angles. This quantity is calculated along a line from the midpoint of the bottom end cap to the midpoint of the top end plate in (r, z) plane. It is found that the pressure profiles show a progressive decrease in the upper part as the apex angles increase. Furthermore, the pressure distributions for $\alpha \neq 0$ are distinct from that plotted in a classical Taylor Couette configuration in two respects. First, the vortices translate in the direction of increasing stator radius. Second, neighboring clockwise and counterclockwise vortices are not reflectively symmetric: vortices with the same sense of rotation as the large-scale circulation are stronger and larger than those with the opposite sense of rotation.
Conclusion

In this work, we have conducted a numerical modeling of the effect of outer cylinder wall alignment on the onset of Taylor vortices. The imperfection is introduced by opening the outer fixed cylinder with a certain angle ranging from 0 up to 12 with respect to the vertical and keeping the inner rotating cylinder as vertical straight wall. Calculations are carried out using a three-dimensional CFD of incompressible viscous flow. The validation is made with experimental results available in literature for the same geometrical parameters. The flow patterns are presented by plotting the distribution of pressure, streamlines and wave number versus cone angle and the Taylor number. It is established that the appearance of the first instability, TVF, for $\alpha \neq 0$ is substantially advanced with respect to classical case, $\alpha = 0$. It is also found that the wave number and size of vortices depend of the apex angle.

Figure 8: Distribution of the pressure along the middle line. In each figure, the left presents the dimensionless mean pressure distribution, and the right illustrates its contour in $(r,z)$ plane.
References


[17] Lalauoa, A., Bouabdallah, A.: On the onset of Taylor vortices in finite-length cavity subject to a radial oscillation motion. Accepted for publication in JAFM.

[18] Adnane, E., Lalauoa, A., Bouabdallah, A.: An experimental study of the laminar-turbulent transition in a tilted Taylor-Couette system subject to free surface effect. Accepted for publication in JAFM.


