ON APPLICATION OF FINITE ELEMENT METHOD FOR APPROXIMATION OF 3D FLOW PROBLEMS

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Abstract

This paper is interested to the interactions of the incompressible flow with a flexibly supported airfoil. The bending and the torsion modes are considered. The problem is mathematically described. The numerical method is based on the finite element method. A combination of the streamline-upwind/Petrov-Galerkin and pressure stabilizing/Petrov-Galerkin method is used for the stabilization of the finite element method. The numerical results for a three-dimensional problem of flow over an airfoil are shown.

Keywords: aeroelasticity, finite element method

1 Introduction

The interaction of flowing fluids and vibrating structures is important in various technical or scientific applications, see, e.g. the monographs [4], [13]. The mathematical simulation of fluid and structure interaction requires to consider the viscous, usually turbulent flow, changes of the flow domain in time, nonlinear behaviour of the elastic structure and to solve simultaneously the coupled system for the fluid flow and for the oscillating structure. The changes of the fluid domain cannot be neglected and the methods with moving meshes must be employed, see e.g. ([5], [11]) .

The subject of our attention is the numerical analysis of the interaction of the incompressible viscous flow with a vibrating airfoil. The numerical analysis of 2D interactions of the incompressible flow with an airfoil was published in [19], [17]. See also [6], where the method allowing the solution of large amplitude flow-induced vibrations of an airfoil with 3 degrees of freedom (3-DOF) was developed and tested. The main difficulty in 2D is the solution of the incompressible flow problem. The approximation of the Navier-Stokes equations for 3D flow problems is even more complicated, see also [20]. The incompressibility is usually treated by using the pressure projection methods originated by Patankar [16], or Chorins artificial compressibility method [2]. The other possibility applied here is to use coupled solution for both pressure-velocity unknowns. For the case of high Reynolds numbers anisotropically refined meshes need to be used in order to capture correctly the boundary layers, wakes, etc. In this paper the attention to the problem of mutual interaction of the airflow with a solid. The motion of the solid is described with the aid of two degrees of freedom (2DOF) (the bending and the torsion modes), which is equivalent to previous two-dimensional (2D) numerical results published, e.g., in [19]. The main attention is paid on the verification of the applied numerical method, which is based on the finite element method fully stabilized using the streamline-upwind/Petrov-Galerkin(SUPG) and the pressure-stabilizing/Petrov-Galerkin (PSPG) stabilization. Here, moreover the grad-div stabilization is included. The 2D and three-dimensional (3D) numerical results are shown and compared.

2 Mathematical model

Flow model. In order to practically treat the motion of the fluid computational domain Ω_t, the Arbitrary Lagrangian-Eulerian (ALE) method is used, see [14]. The ALE mapping $\mathcal{A} = \mathcal{A}(\xi, t) = \mathcal{A}_t(\xi)$ defined for all $t \in (0, T)$ and $\xi \in \Omega_0^{\mathrm{ref}} = \Omega_0$ is assumed to be diffeomorphism of $\overline{\Omega_0}$ onto $\overline{\Omega_t}$ at any $t \in (0, T)$. The domain velocity $\boldsymbol{w}_D(x, t)$ is then defined by $\boldsymbol{w}_D(x, t) = \frac{\partial \mathcal{A}}{\partial t}(\xi, t)$, where

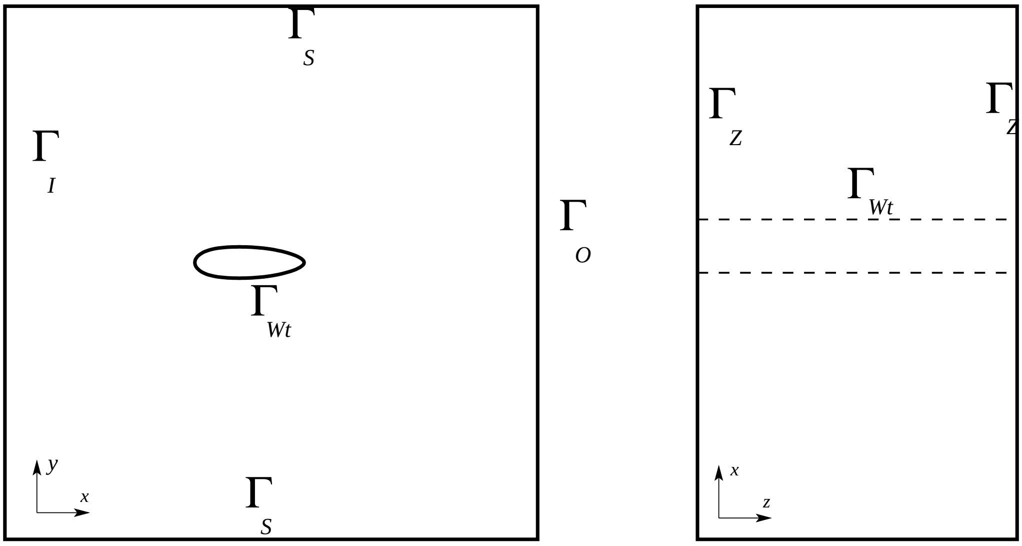

Figure 1: The sketch of the computational domain Ω_t shown in xy and xz planes. The mutually disjoint parts of its boundary $\partial\Omega$ are shown.

$x = \mathcal{A}(\xi, t)$, $x \in \Omega_t$, $\xi \in \Omega_0$. The time derivative with respect to the reference configuration Ω_0^{ref} is called the ALE derivative, denoted by $D^{\mathcal{A}}/Dt$ and satisfies (see [19], [14])

$$\frac{D^{\mathcal{A}} f}{Dt}(x,t) = \frac{\partial f}{\partial t}(x,t) + \boldsymbol{w}_D(x,t) \cdot \nabla f(x,t). \tag{1}$$

The incompressible viscous flow in the computational domain $\Omega \subset \mathbb{R}^3$ is governed by the Navier-Stokes equations written in the ALE form

$$\frac{D^{\mathcal{A}} \boldsymbol{u}}{Dt} + ((\boldsymbol{u} - \boldsymbol{w}_D) \cdot \nabla)\boldsymbol{u} + \nabla p - \nabla \cdot (2\nu \boldsymbol{S}(\boldsymbol{u})) = 0, \qquad \nabla \cdot \boldsymbol{u} = 0, \tag{2}$$

where $\boldsymbol{u} = \boldsymbol{u}(x,t)$ denotes the velocity vector, $\boldsymbol{u} = (u_1, u_2, u_3)$, $p = p(x,t)$ denotes the kinematic pressure, ν is the constant kinematic viscosity (i.e. the viscosity divided by the constant fluid density ρ), t denotes time, $\boldsymbol{S} = \frac{1}{2}(\nabla \boldsymbol{u} + \nabla^T \boldsymbol{u})$ is the rate of the strain tensor whose components are given by

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right).$$

System of equations (2) is equipped with the initial

$$\mathbf{u}(x,0) = \mathbf{u}^0(x) \qquad x \in \Omega, \tag{3}$$

and with boundary conditions

$$\begin{array}{llll} \text{a)}\ \boldsymbol{u} = \boldsymbol{u}_D & \text{on } \Gamma_I, & \text{b)}\ \boldsymbol{u} = \boldsymbol{0} & \text{on } \Gamma_{Wt}, \\ \text{c)}\ -p\boldsymbol{n} + \nu \boldsymbol{S}\boldsymbol{n} = 0 & \text{on } \Gamma_O, & \text{d)}\ \boldsymbol{u} \cdot \boldsymbol{n} = 0, & (-p\mathbf{n} + \nu \boldsymbol{S}\mathbf{n}) \times \boldsymbol{n} = 0 \text{ on } \Gamma_S, \end{array} \tag{4}$$

where $\mathbf{n}$ denotes the unit outward normal vector to the Lipschitz continuous boundary $\partial\Omega$. The boundary condition (4c) is a modification of the so-called 'do-nothing' boundary condition, cf. [1].

Structure motion. The flow model is coupled with the structure model representing the flexibly supported airfoil. The airfoil can be vertically displaced by h (downwards positive) and rotated by angle α (clockwise positive). In this paper only the small displacements of the structure are taken into account and the linear equations of motion read

$$m\ddot{h} + S_\alpha \ddot{\alpha} + k_h h = -L(t), \qquad S_\alpha \ddot{h} + I_\alpha \ddot{\alpha} + k_\alpha \alpha = M(t). \tag{5}$$

where m is the mass of the airfoil, S_α is the static moment around the elastic axis (EA), and I_α is the inertia moment around EA. The parameters k_h and k_α denote the stiffness coefficients. On the right-hand side the aerodynamical lift force $L(t)$ and aerodynamical torsional moment $M(t)$ are involved, which satisfy

$$L = -\int_{\Gamma_{Wt}} \sigma_{2j} n_j \, dS, \qquad M = \int_{\Gamma_{Wt}} \varepsilon_{3ij} r_i \sigma_{jk} n_k \, dS, \tag{6}$$

where

$$\sigma_{ij} = \rho \left[-p\delta_{ij} + 2\nu S_{ij} \right], \tag{7}$$

ε_{3ij} is the Levi-Civita symbol, $r_i = x_i - x_i^{\mathrm{EA}}$ and x^{EA} is the position of the EA at the time instant t.

3 Numerical approximation

In this section the time discretization of the flow problem is introduced, linearized and the resulting problem is spatially approximated by the finite element method based on continuous piecewise linear (or tri-linear) functions is used for approximation of both the velocity and the pressure. The application of the finite element method for the incompressible Navier-Stokes equations needs to overcome two difficulties. First, the velocity-pressure finite element pair needs to be properly chosen in order to guarantee the stability of the scheme, see, e.g., [21], or the PSPG can be used to overcome the instability, see [8]. The second source of the instability is the dominating convection flows. In this case the SUPG method is applied, cf. [15], [12], [7].

3.1 Time discretization

First, we consider a partition $0 = t_0 < t_1 < \cdots < T,\ t_k = k\Delta t$ of the time interval $[0, T]$ with a constant time step $\Delta t > 0$, approximate $\boldsymbol{u}(t_n)$, $p(t_n)$, $\alpha(t_n)$, $h(t_n)$, $\dot\alpha(t_n)$, $\dot h(t_n)$ and $\boldsymbol{w}_D(t_n)$ by $\boldsymbol{u}^n$, p^n, α^n, h^n, $\dot\alpha^n$, $\dot h^n$ and $\boldsymbol{w}_D^n$, respectively. Here, the attention is paid only to the discretization on a fixed time level t_{n+1}.

The ALE derivative in (2) is then approximated by second order backward difference formula (BDF2), i.e.

$$\frac{D^{\mathcal{A}}\boldsymbol{u}}{Dlt}\Big|_{t=t_{n+1}} \approx \frac{3\boldsymbol{u}^{n+1} - 4\tilde{\boldsymbol{u}}^n + \tilde{\boldsymbol{u}}^{n-1}}{2\Delta t}, \tag{8}$$

where $\tilde{\boldsymbol{u}}^i(x) = \boldsymbol{u}^i(\mathcal{A}(t_i, \xi))$ with $x = \mathcal{A}(t_{n+1}, \xi)$ is the transformation of the velocity from the domain Ω_{t_i} on the domain $\Omega_{t_{n+1}}$. In order to linearize the problem, the convective term is linearized by

$$((\boldsymbol{u} - \boldsymbol{w}_D)\cdot\nabla)\boldsymbol{u}\Big|_{t_{n+1}} \approx (2\tilde{\boldsymbol{u}}^n - \tilde{\boldsymbol{u}}^{n-1} - \boldsymbol{w}_D^{n+1})\cdot\nabla\boldsymbol{u}^{n+1}, \tag{9}$$

where we shall write $\overline{\boldsymbol{w}}^{n+1} = 2\tilde{\boldsymbol{u}}^n - \tilde{\boldsymbol{u}}^{n-1} - \boldsymbol{w}_D^{n+1}$.

The system of equations (5) is transformed to the first order system and discretized in time using the BDF2, i.e. the following approximations are used

$$\dot\alpha(t_{n+1}) \approx \frac{3\alpha^{n+1} - 4\alpha^n + \alpha^n}{2\Delta t}, \qquad \dot h(t_{n+1}) \approx \frac{3h^{n+1} - 4h^n + h^n}{2\Delta t},$$
$$\ddot\alpha(t_{n+1}) \approx \frac{3\dot\alpha^{n+1} - 4\dot\alpha^n + \dot\alpha^n}{2\Delta t}, \qquad \ddot h(t_{n+1}) \approx \frac{3\dot h^{n+1} - 4\dot h^n + \dot h^n}{2\Delta t}.$$

Further, the aerodynamical lift force and the aerodynamical moment in equations (5) are extrapolated from the previous time levels, i.e. the approximations $L^n, M^n, L^{n-1}, M^{n-1}$ are assumed to be computed using (6) and the values of $\boldsymbol{u}^n, p^n, \boldsymbol{u}^{n-1}, p^{n-1}$. Then we extrapolate in (5) the aerodynamical lift force and the aerodynamical moment by

$$L(t_{n+1}) \approx 2L^n - L^{n-1}, \qquad M(t_{n+1}) \approx 2M^n - M^{n-1}.$$

3.2 Spatial approximation

Now, the system of equation (2) is time discretized and linearized with the aid of (8) and (9), respectively. Then, the equations are formulated weakly and the solution is sought on the couple of finite element spaces $\boldsymbol{W}_h \subset \boldsymbol{H}^1(\Omega^{n+1})$ and $Q_h \subset L^2(\Omega^{n+1})$ for the approximation of the velocity components and pressure. The domain Ω is assumed to be a polyhedral domain and the finite element spaces are defined using an admissible triangulation $\mathcal{T}_h$ of the domain Ω, cf. [3], such that $\overline{\Omega} = \bigcup_{K\in\mathcal{T}_h} K$, $\mathcal{T}_h$ is formed by a finite number of closed hexahedral, tetrahedral, pyramidal or prism elements. Further, on each of reference elements K the local set of functions is denoted by P_K. For the reference hexahedral element the tri-linear functions are used whereas for the reference tetrahedral element the space P_K contains linear functions on K. The consistent definition of P_K on the pyramidal or the prism elements can be found in [22]. The finite element spaces are then defined by

$$Q_h = \left\{\varphi \in C(\overline{\Omega}) \,:\, \varphi\Big|_K \in P_K, \forall K \in \mathcal{T}_h\right\},$$

$$\boldsymbol{X}_h = \{\boldsymbol{\varphi} \in \boldsymbol{W}_h \,:\, \boldsymbol{\varphi} = 0 \text{ on } \Gamma_I \cup \Gamma_{Wt} \cup \Gamma_Z,\ \boldsymbol{\varphi}\cdot\boldsymbol{n} = 0 \text{ on } \Gamma_S\},$$

and $\boldsymbol{W}_h = [Q_h]^3$. In order to introduce the stabilized weak formulation we start with the definition of the Galerkin terms for any $U = (\boldsymbol{u}, p) \in \boldsymbol{W}_h \times Q_h$, $V = (\boldsymbol{\varphi}, q) \in \boldsymbol{X}_h \times Q_h$ by

$$\begin{aligned}
a(U,V) &= \frac{3}{2\Delta t}(\boldsymbol{u},\boldsymbol{\varphi})_\Omega + (\nu \boldsymbol{S}(\boldsymbol{u}), \boldsymbol{S}(\boldsymbol{\varphi}))_\Omega + (\overline{\boldsymbol{w}}^{n+1}\cdot\nabla\boldsymbol{u},\boldsymbol{\varphi})_\Omega - (p,\nabla\cdot\boldsymbol{\varphi})_\Omega + (\nabla\cdot\boldsymbol{u},q)_\Omega,\\
f(\boldsymbol{u},\boldsymbol{\varphi}) &= \frac{1}{2\Delta t}(4\tilde{\boldsymbol{u}}^n - \tilde{\boldsymbol{u}}^{n-1},\boldsymbol{\varphi})_\Omega,
\end{aligned}$$

where by $(\cdot,\cdot)_\Omega$ the scalar product in $L^2(\Omega)$ or $\boldsymbol{L}^2(\Omega)$ is denoted. Further, the SUPG/PSPG and div-div stabilization terms are used defined by

$$\begin{aligned}
\mathcal{L}(U,V) &= \sum_{K\in\mathcal{T}_h} \delta_K \Big(\frac{3\boldsymbol{u}}{2\Delta t} - \nabla\cdot(\nu\boldsymbol{S}(\boldsymbol{u})) + (\overline{\boldsymbol{w}}^{n+1}\cdot\nabla)\boldsymbol{u} + \nabla p, (\overline{\boldsymbol{w}}^{n+1}\cdot\nabla)\boldsymbol{\varphi} + \nabla q\Big)_K,\\
\mathcal{F}(V) &= \sum_{K\in\mathcal{T}_h} \delta_K \Big(\frac{4\tilde{\boldsymbol{u}}^n - \tilde{\boldsymbol{u}}^{n-1}}{2\Delta t}, (\overline{\boldsymbol{w}}^{n+1}\cdot\nabla)\boldsymbol{\varphi} + \nabla q\Big)_K, \qquad \mathcal{P}(U,V) = \sum_{K\in\mathcal{T}_h} \tau_K(\nabla\cdot\boldsymbol{u},\nabla\cdot\boldsymbol{\varphi})_K.
\end{aligned}$$

where $(\cdot,\cdot)_K$ denotes the scalar product in $L^2(K)$ or $\boldsymbol{L}^2(K)$. The choice of the parameters δ_K and τ_K is carried out according to [7] or [18] on the basis of the local element length h_K, i.e.

$$\tau_K = \nu\left(1 + Re^{\text{loc}} + \frac{h_K^2}{\nu\cdot\Delta t}\right), \qquad \delta_K = \frac{h_K^2}{\tau_K}, \qquad Re^{\text{loc}} = \frac{h_K \|\overline{\boldsymbol{w}}^{n+1}\|_K}{2\nu}. \tag{10}$$

Then the *stabilized discrete problem* at a time instant $t = t^{n+1}$ reads: Find $U = (\boldsymbol{u}, p) \in \boldsymbol{W}_h \times Q_h$, $p := p^{n+1}$, $\boldsymbol{u} := \boldsymbol{u}^{n+1}$, such that $\boldsymbol{u}$ satisfies approximately the conditions (4 a-b) and

$$a(U,V) + \mathcal{L}(U,V) + \mathcal{P}(U,V) = f(V) + \mathcal{F}(V),$$

holds for all $V = (\boldsymbol{\varphi}, q) \in \boldsymbol{X}_h \times Q_h$. The solution of the arising system of linear equations is realized by a preconditioned iterative method based on inexact LU-factorization, see [**?**].

4 Numerical results

The described method was applied for an approximation of channel flow over NACA 0015 and NACA 0012 airfoils. The channel geometry was chosen according to available measurements, see [10] or [9], where the mutual interaction of the flowing air caused lead to the aeroelastic instability of flutter type. We consider the depth of the domain to be 180 mm (z-direction in Figure 1), the height of the domain (y-direction in Figure 1) to be equal to 210 mm which is the height of the experimental channel of the measurement. The length of the computational domain was chosen to be 5 times the cord of the airfoil. The airfoil cord was equal to $c = 18$ mm. The computations were performed on hexahedral meshes. The air viscosity was considered to be equal

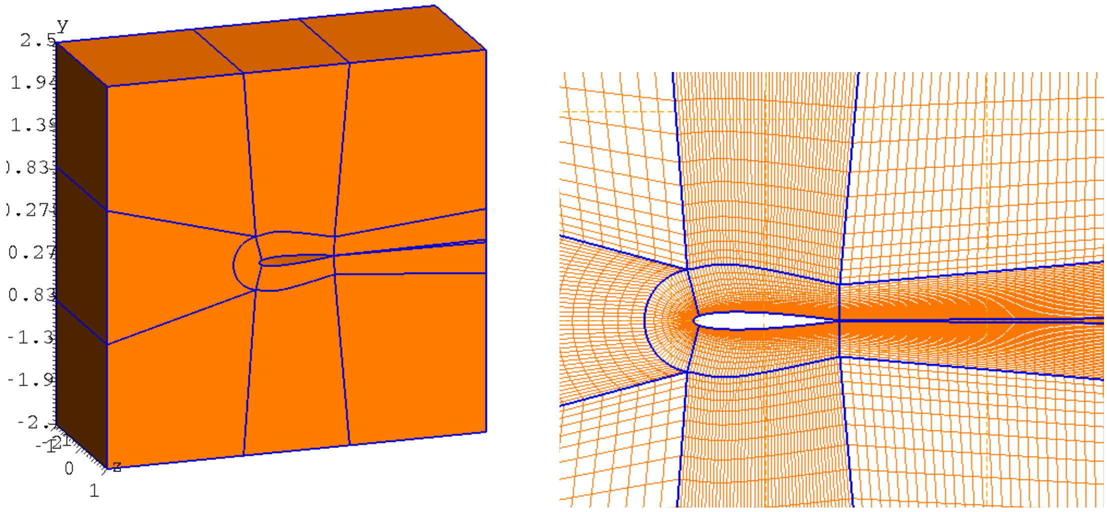

Figure 2: The computational domain and the mesh detail around the airfoil shown in the xy-plane.

to $\nu = 1.5 \times 10^{-5}\,\mathrm{m}^2 \,/\, \mathrm{s}$. The inlet velocity was set equal to $U_\infty = 8.33\,\mathrm{m} \,/\, \mathrm{s}$. The Reynolds number based on the airfoil cord is then equatl to $Re = 10^4$.

For the numerical simulation two grids were used. First, the coarse grid consisting of approximately 200000 hexahedral elements was used, where the discrete system with approximately 900000 unknowns was solved using the iterative solver based on the incomplete blockwise LU factorization. Second, the grid with approximately 700000 hexahedral elements was used leading to the system with approximately 2.8×10^6 unknowns. The iterative solver was successful in both cases, although the numerical solution on the fine grid required much more computer time (as expected). The further improvement, optimization and parallelization of the iterative solver needs to be done.

The solution of the 3D problem was compared to the numerical approximation of the flow in 2D domain, where the grid consisting of the approximately 10000 quadrilateral elements was used. For the 2D problem the solution was realized using the direct solver. The fluid 3D domain is shown shown in Figure 2. The same grid (a slice of the 3D grid) was used for the computation of the 2D flow problem. The results in terms of flow velocity field for 2D and the central plane of 3D computations nearby the airfoil surface are shown in Figure 3. The flow pattern is very similar for both case. Particularly, the flow velocity close to the airfoil surface is almost identical. A difference in the flow field can be observed, which seems to be caused first by three-dimensional character of the flow and second by the channel walls considered in the 3D simulations. Similarly, the pressure distribution is compared in Figure 4 again with very similar distribution close to the airfoil surface. Similarly as slightly higher flow velocities in the central plane are observed for the 3D results, see Figure 5, also a small difference in the pressure field can be identified, see Figure 6.

5 Conclusion

This paper focused on the problem of numerical approximation of the incompressible flow with a flexibly supported airfoil. A model problem of channel flow around vibrating NACA 0015 airfoil was considered and mathematically described. The numerical method based on the finite element method was succesfully used for the approximation of the flow problem. This numerical method is the extension of the stabilized finite element method applied succesfully for 2D simulations. Here, however the method was tested and verified by application on a channel flow over the NACA 0012 airfoil. The results were compared to 2D resuls. Furthermore, the numerical approximation based on fully stabilized finite element method was presented and the numerical results for a three-dimensional problem of flow over an airfoil were shown.

Furthermore, the iterative method for solution of the discrete problem in 3D was succesfully used. The iterative method based on the blockwise incomplete LU factorization was used. The

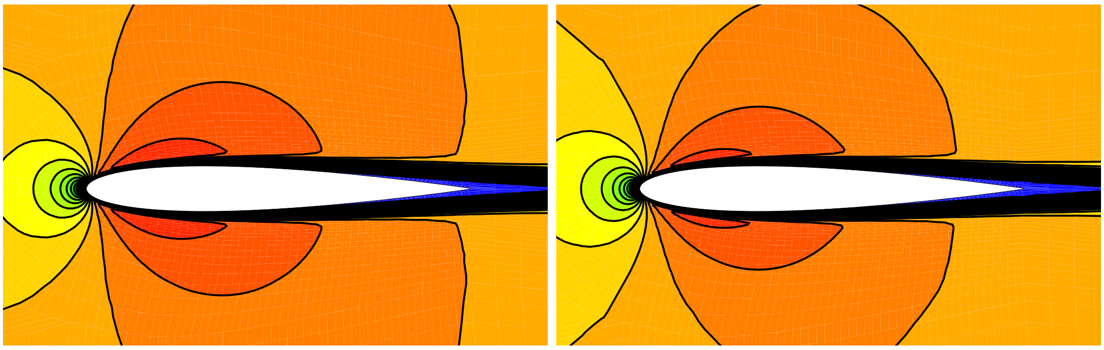

Figure 3: The comparison of the flow velocity pattern for 3D (on the left) and 2D (on the right) computations.

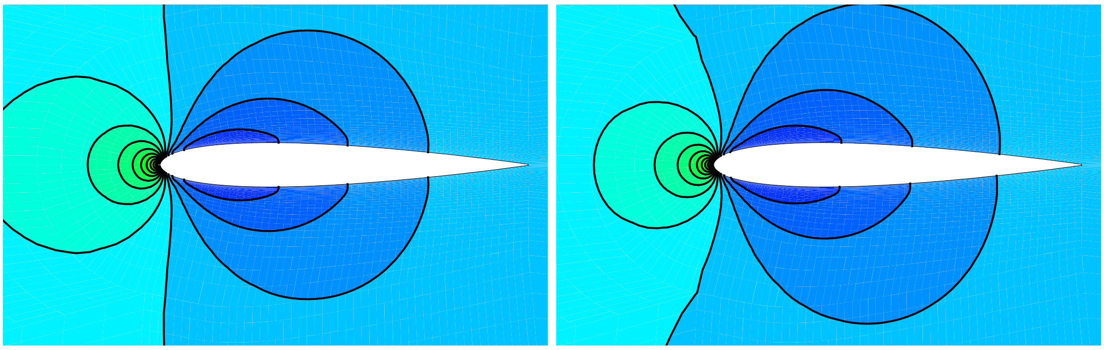

Figure 4: The comparison of the pressure distribution for 3D (on the left) and 2D (on the right) computations.

method was found to be efficient, but for large meshes the convergence was slow (as expected). The improvement of the iterative method will be a subject of future work.

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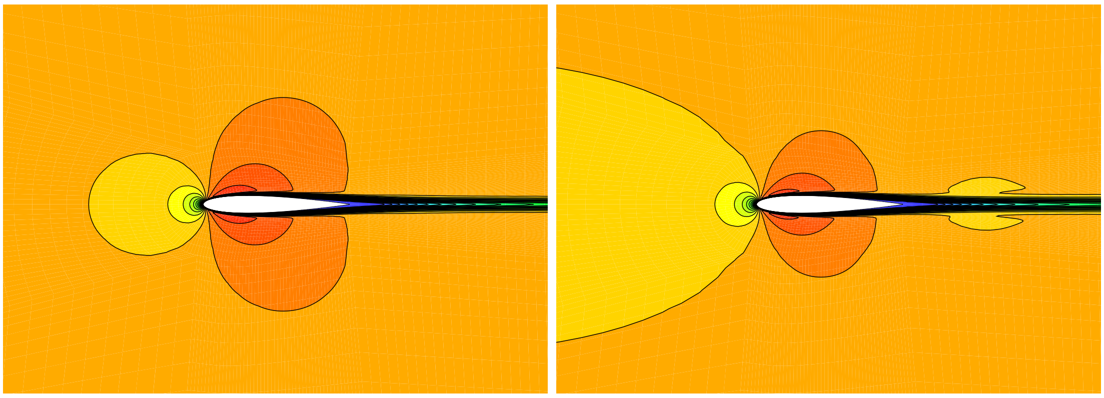

Figure 5: The comparison of the flow velocity pattern for the central plane of 3D (on the left) and 2D (on the right) computation.

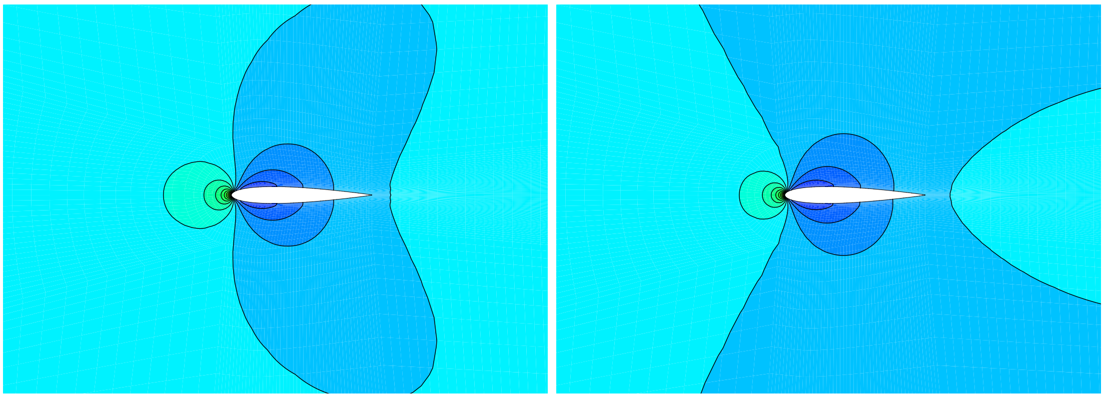

Figure 6: The comparison of the pressure distribution for the central plane of 3D (on the left) and 2D (on the right) computation.

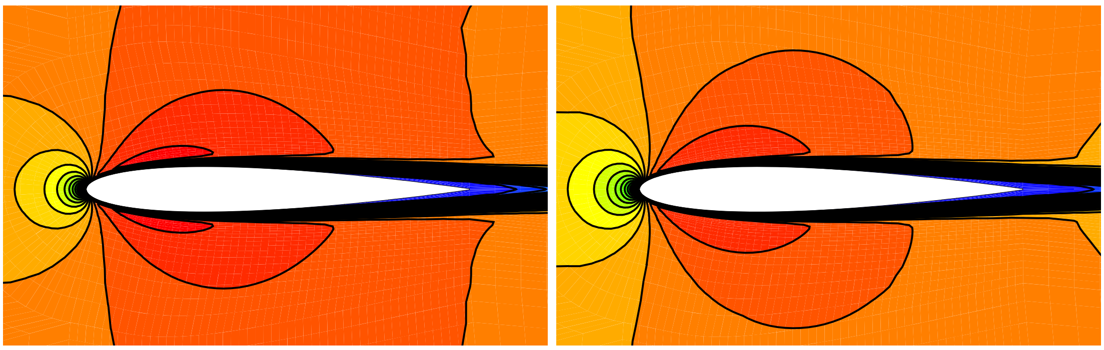

Figure 7: The magnitude of flow velocity patterns for the z-plane $z = 0.9$ (on the left) and $z = 0.8$ (on the right).

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