INFLUENCE OF THE ANGULAR EFFECT ON THE HEAT TRANSFER PROCESS IN GRAËTZ HEAT EXCHANGER

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Abstract

We modeled the process to establish the properties related to heat transfer involved the modified Graëtz system in MHD. Thus using a computational code based on finite volume method (Fluent CFD) sought to extend our study by generalizing the problem to angular effects when plates are no longer parallel (classical system) and become a convergent device (modified system) defined by the inclination angle α, bounded between the device plate and the axis of the flow system. The advantage of such modification will directly affect the distribution of the velocity field, temperature with or without a magnetic field effect. Under these conditions, we can expect an improvement in the transfer process by highlighting the increasing Nusselt number.

Keywords: heat transfer, Graëtz problem, Nusselt number

1 Introduction

This work is aimed to study the Graëtz heat exchanger subjected to the simultaneous influence of the temperature and the magnetic fields in order to evaluate the heat transfer performance under the imposed conditions which are close to industrial activities.

In heat transfer techniques, the Graëtz method is an essential control process to influence energy production operations. We find this type of process mainly in several activities as steel, plastic materials and energy production. It is well known that the fact of having a properly sized and suitable process can achieve very significant gains in performance. Laminar heat transfer through a parallel plates subject to uniform wall temperature is studied by taking into account both viscous dissipation and fluid axial heat conduction in a finite thermal entrance region.

Concerning laminar forced convection heat and mass transfer in cylindrical and parallel plates at steady state in a bounded channel with ignoring axial heat conduction is well known Graëtz problems [1][2]. An extended Graëtz problem [3][4] is the influence of axial heat conduction being considered, particularly, for the low Prandtl number fluids such as liquid metals. The problems extended to two or more contiguous phases with coupling through conjugated conduction-convection conditions at the boundaries by associating the governing equations in each phase are the conjugated Graëtz problems [5][6].

The established laminar flow of an electrically conducting incompressible fluid or liquid metal flows through parallel plates under a uniform magnetic field imposed perpendicular to the walls, was studied by Hartmann and Lazarus and was extended by T.G. Cowling [7] this flow is called Hartmann flow. R. Siegel and al [8] analyzed the temperature distribution in the thermal entrance region under the influence of a constant wall heat flux. V. Javeri [9] was investigating the laminar MHD heat transfer coupled with the forced convection in thermal entrance region, confined in a parallel plate channel with boundary condition of the third kind (Robin boundary conditions)[9] by neglecting the heat generation. He solved the energy equation by applying the Galerkin-Kantorowich method of variationnal calculus. J. Lahjomri [10] studied analytically the thermal development of laminar flow through a parallel plate channel, by taking into account the transverse variation in fluid temperature, viscous dissipation, Joule heating and axial heat conduction.

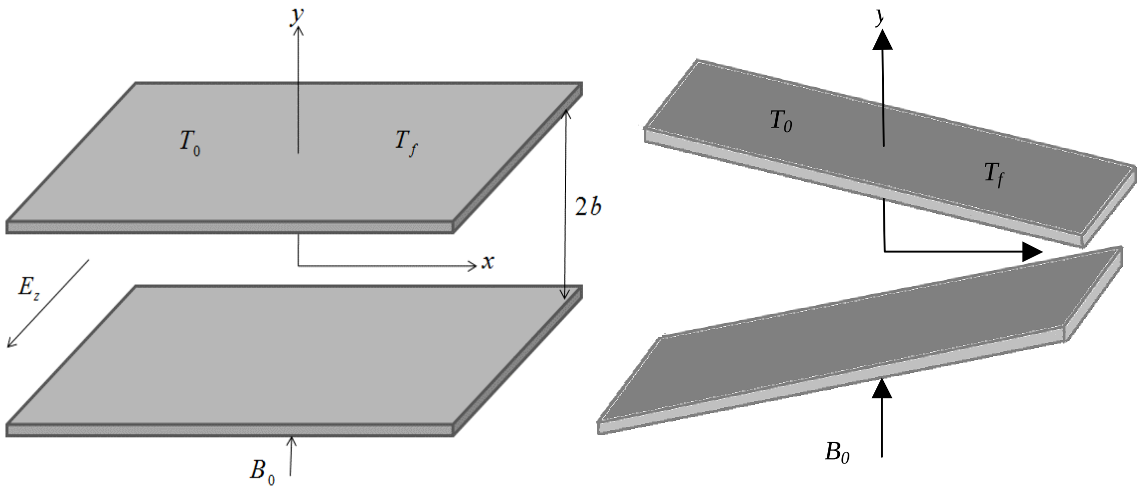
Our principal attempt is the improvement of the heat exchanges by choosing adequately the geometrical characteristics of the Graëtz device in order to better control the entrance region.

Therefore, we assume the technical conditions close to previous authors, by introducing a parameter, the inclination angle α, what defines a new device with convergent plates but by keeping the same aspect ratio Γ[10].

The present problem corresponds to the numerical study of transverse magnetic field influence on the fundamental extended Graëtz problem. The main objective is to determine the temperature field and Nusselt number in the thermal entrance region.

2 Formulation of the problem

We consider the flow of an incompressible and electrically conducting liquid metal, flowing at a mean velocity $\bar{U}_m$ following the axial direction x. It is confined between two parallel plates separated by a distance equal to 2b, subjected to the transversal magnetic field action B_0 parallel to y direction. The plates constituting the channel are heated with prescribed wall heat flux Q_W. It is assumed that the channel considered is maintained at an uniform temperature T_0 for x<0 and T_f for x>0 (fig 1).



(a) (b)

Figure 1: Schematic diagram: classical system (a) and modified system (b)

In particular, one proposes to study the Graëtz heat exchanger (Fig 1) submitted to the simultaneous influence of the temperature and magnetic fields to assess the performance of heat transfer in the conditions imposed which are close to the industrial processing.

In order to simplify the study, we are led to consider the following assumptions as

1. The physical properties of the fluid that is the liquid silicon are constants.

2. It is supposed that the viscous flow is steady laminar MHD, and hydrodynamically fully developed in the whole length of the heat exchanger.

3. Fixed temperatures are imposed on the two Graëtz thermal regions.

4. The free convection caused by the temperature difference is negligible but the effects of the viscous dissipation, Joule heating and axial heat conduction are taken into account.

5. The flow is one-way so as to the fluid runs out according to the axial direction.

6. One limits the investigation to the small angle of modified system.

3 Governing Equations

The magnetic field considered in the studied system generates in the fluid or the liquid metal electroconductor mechanical and thermal effects whose intensity depends primarily on the characteristics of the magnetic field applied.

The basic equations governing the electric behavior of the fluid or the conducting liquid metal subjected to the magnetic induction field are mainly partial differential equations using the continuity equation, Navier-stokes equations and Maxwell equations.

3.1. Hydrodynamics equations

Taking account of the stated assumptions mentioned previously, one leads to the following formulation [10] as

Continuity

$$\frac{\partial \bar{U}_x}{\partial x} + \frac{\partial \bar{U}_y}{\partial y} = 0 \tag{1}$$

Momentum

$$\frac{3}{2}\mu\left[\frac{\partial^2 \bar{U}_x}{\partial x^2} + \frac{\partial^2 \bar{U}_x}{\partial y^2}\right] = \frac{\partial p}{\partial x} + \left(j_y B_z - j_z B_y\right) \tag{2}$$

$$\frac{\partial p}{\partial y} = \rho g + \left(j_x B_z - j_z B_x\right) \tag{3}$$

Energy

$$\rho C_p \bar{U}_x(y)\left(\frac{\partial T_i}{\partial x}\right) = k\left[\frac{\partial^2 T_i}{\partial x^2} + \frac{\partial^2 T_i}{\partial y^2}\right] + \mu\left(\frac{d\bar{U}_x(y)}{dy}\right)^2 + \frac{j_z^2}{\sigma} \tag{4}$$

With the associated boundary conditions in section (3.3).

3.2. Electromagnetism equations

Electromagnetism is characterized mainly by the Maxwell's equations and the Ohm generalized law.

Knowing that the Ohm law connects the electrical current density to the magnetic field, one can write it as follows

$$j_z^2 = \sigma^2 \left(E_z + \bar{U}_x B_y \right)^2 \tag{5}$$

In this type of flow, one sees that $\bar{U}_x$ is related to y only and B_0 is a constant from where the system is linear in $\bar{U}_x$ and B_0.One is brought to define the velocity profile assuming that the flow is entirely developed, so that the resolution of the equations (2) and (3) which will give the following result,

$$\bar{U}_x(y) = \bar{U}_m \frac{\left(\mathrm{chHa} - \mathrm{chHa}\,\frac{y}{b} \right)}{\left(\mathrm{chHa} - \frac{\mathrm{shHa}}{\mathrm{Ha}} \right)} \tag{6}$$

For simplicity, the governing equations are writing in dimensionless form using the following reduced variables and functions given by

$$\xi = \frac{x}{b\mathrm{Pe}},\ \eta = \frac{y}{b},\ \theta_i(\xi,\eta) = \frac{T_i - T_f}{T_0 - T_f},\ \bar{U}(\eta) = \frac{\bar{U}_x}{\bar{U}_m} \tag{7}$$

By introducing dimensionless variables (7) in the energy equation, we obtain the linearized governing equation,

$$\bar{U}(\eta)\frac{\partial \theta_i}{\partial \xi} = \frac{\partial^2 \theta_i}{\partial \eta^2} + \frac{1}{\mathrm{Pe}^2}\frac{\partial^2 \theta_i}{\partial \xi^2} + B_r D(\mathrm{Ha},\eta) \tag{8}$$

Where,

$$D(\mathrm{Ha},\eta) = \gamma^2 \mathrm{Ha}^2 \mathrm{ch}(2\mathrm{Ha}\eta) + \mathrm{Ha}^2(\chi + \gamma \mathrm{chHa})^2 - 2\mathrm{Ha}^2\gamma(\gamma \mathrm{chHa} + \chi)\mathrm{chHa}\eta,$$

$$\text{with } \gamma = \frac{1}{(\mathrm{chHa} - \frac{\mathrm{shHa}}{\mathrm{Ha}})}$$

In the equation cited above, we note the presence of the characteristic numbers involving the Brinkman number (B_r), the Peclet number (Pe) and the Hartmann number (Ha). These dimensionless numbers are defined as

$$\mathrm{Br} = \frac{\mu \bar{U}_m{}^2}{bQ_w},\quad \mathrm{Pe} = \frac{3\bar{U}_m b}{2\alpha},\ \text{and } \mathrm{Ha} = B_0 b\sqrt{\sigma/\mu}$$

3.3. Boundary conditions

The boundary conditions associated to the both regions of Graëtz process defined previously, are established to solve the thermal equation and it follows that

$$\left(\frac{\partial \theta_i}{\partial \eta} \right)_{\eta=0} = 0,\ \text{with } i = 1, 2$$

$$\text{For } \forall \xi \le 0 \text{ and } \eta = 1:\ \theta_1(\eta,\xi) = 1 \text{ and } \theta_2(\eta,\xi) = 0\ \ (\forall \xi \succ 0)$$

$$\text{For } \xi = 0 \text{ and } 0 \le \eta \prec 1:\ \theta_1(\eta,\xi) = \theta_2(\eta,\xi) \text{ and } \left(\frac{\partial \theta_1}{\partial \xi} \right) = \left(\frac{\partial \theta_2}{\partial \xi} \right)$$

4. Results and discussion

In order to study by numerical simulation, we used Fluent software part of the Ansys. The governing equations are discretized and solved numerically using this code based on the finite volumes method.
The Gambit software is used to build the geometry, generate the meshing, and define the boundary conditions associated to the considered system (interfaces, stiff walls, fluid, etc.)
In our investigation, a test of the adequate meshing is made by means of several tries to fix the appropriate number of nodes and optimize the time and the space of storage before starting the calculation. Consequently, we used a structured quadrature meshing composed of 20480 Cells, 41344 faces and 20865 nodes for devices, for classic and modified systems.

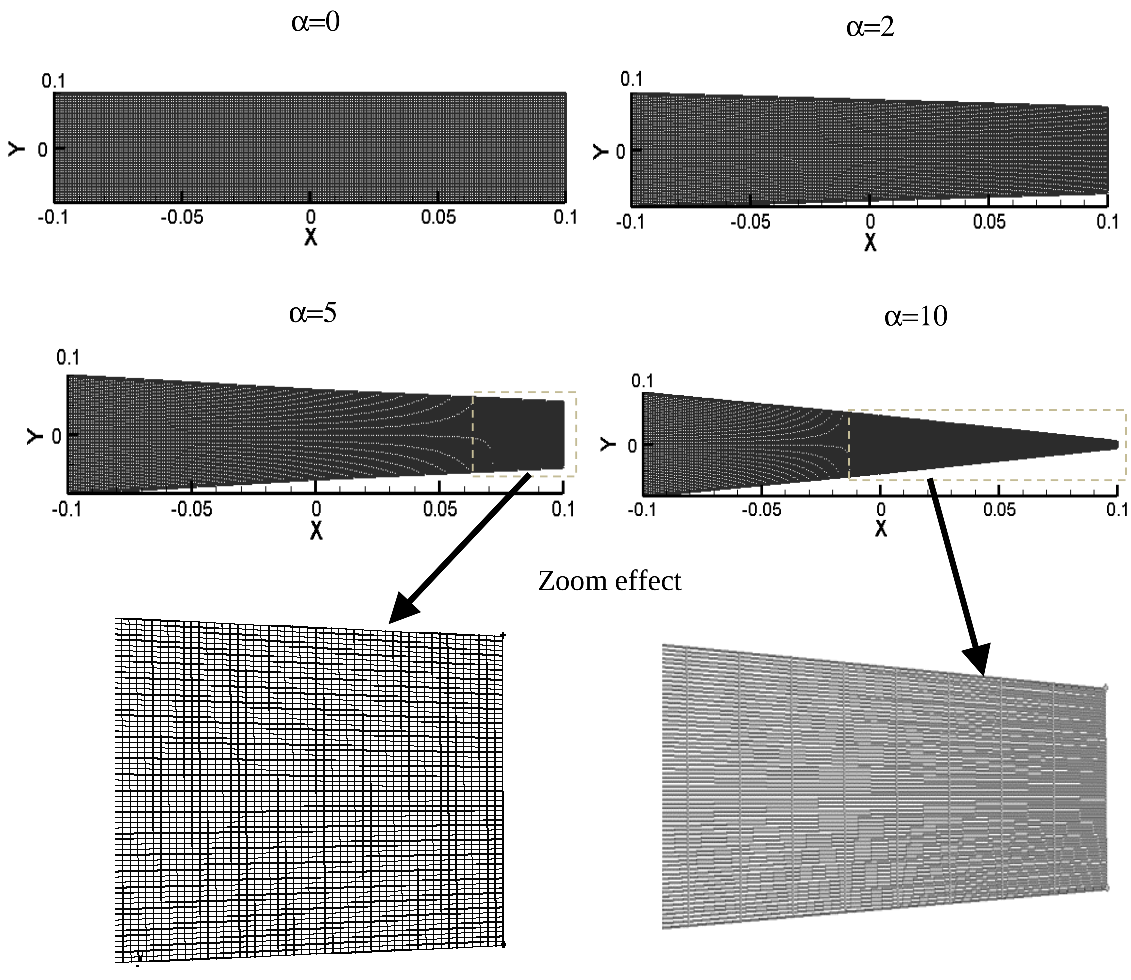


Figure 2: Meshing of the domain plan according to various angles α

4.1. Angular effect

For a given Reynolds number defined under the onset of laminar-turbulent transition regime, the flow of the liquid silicon within the Graëtz device is laminar. For that purpose, we make simulations on the Graëtz device for various inclination angles knowing that Re is maintained constant around 2000 and the aspect ratio $\Gamma = L/b$ describing the channel being equal to 5 as in [10].

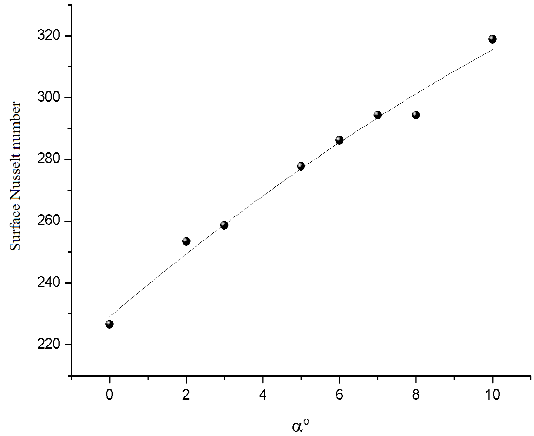


Figure 3: Surface Nusselt number variation according to the inclination angles α, at Re = 2000 and Γ = 5

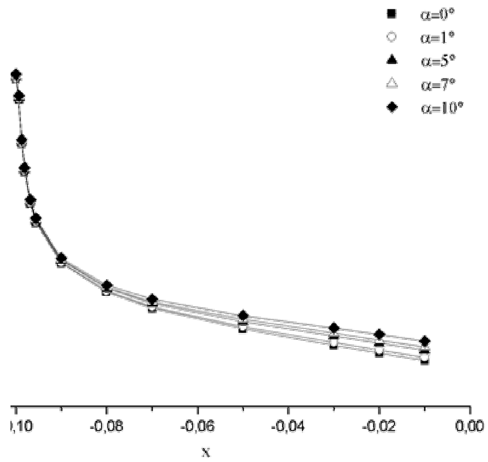


Figure 4: Local Nusselt number Nu_x evolution along the entrance region for different inclination angles α, at Re = 2000 and Γ = 5

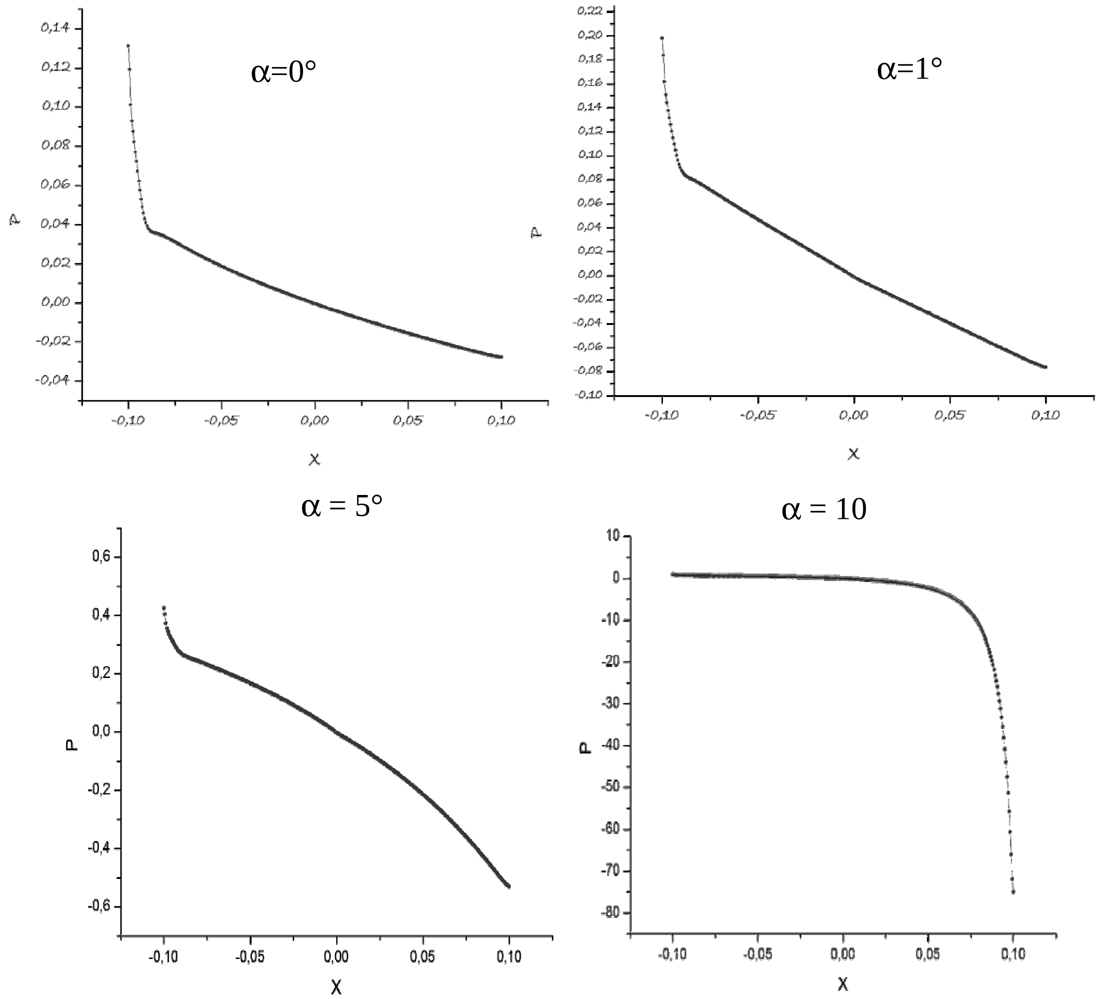


Figure 5: Pressure evolution along the channel for different inclination angles α, at Re = 2000 and ξ = -0.01

It is important to underline that the heat exchanges by convection are more important in such a way we increase the inclination angle α which it serves mainly to bring up the flow velocity to the vicinity of the upstream region. Consistently, the angular effect increases the heat transfer by convection.
The pressure field, in the entrance region of classical Graëtz device, decreases quickly until the lower extremity of the channel while in the presence of the modified one it stabilizes along the entrance region for abruptly decreasing towards the upper extremity of the flow system.

4.2. Forced convection

The forced convection due to the velocity of the flow has a determining role within the heat exchangers and the energy production system heat transfers.

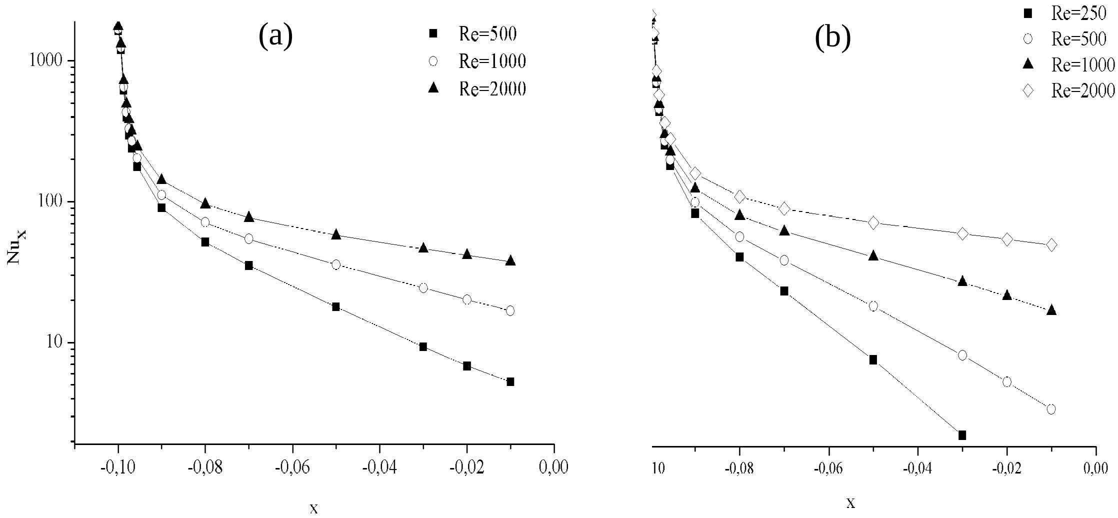


Figure 6: Local Nusselt number Nu_x evolution along the entrance region versus Reynolds number for classical system (a), modified system (b).

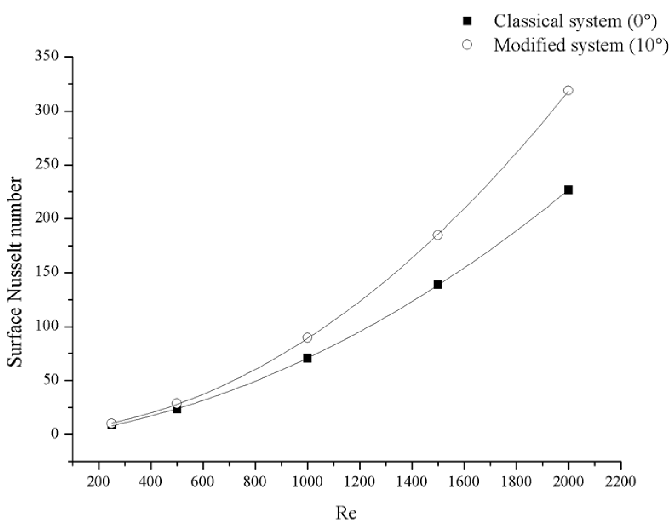


Figure 7: Surface Nusselt number Nu variation according to the Reynolds number in the classical and modified systems

It is known that the axial heat conduction varies conversely with the forced convection, and is characterized by the Peclet number which is defined by the product of two other dimensionless numbers: the Prandtl and the Reynolds numbers such as $P_e = Re.P_r$
One proposes to carry out simulations by varying the Reynolds number from Re = 250 to Re = 2000 while the Peclet number will be included in the range $3 \leq Pe \leq 23$.
By analyzing the curves obtained for the Nusselt number Nu defined by the rate of convective transfer compared to conduction, one sees that Nu is very sensitive to the velocity which flows along the Graëtz channel, it varies linearly and proportionally with the Reynolds number. It behaves into decreasing exponential law along the thermal entrance region and increases under the forced convection effect.
The maximal and minimal values giving the Nusselt number describing the entrance region corresponding to the studied devices, in terms of the Reynolds number are presented in table 1

	Classical system		Modified system	
	Nu_{max}	Nu_{min}	Nu_{max}	Nu_{min}
Re=500	1634	37.8	1741	371
Re=1000	1677	45. 9	1783	44
Re=2000	1755	62.8	1858	68.8

Table 1: Forced convection effect on the Nusselt number Nu

Due to the evolution of the Nusselt number, one can see the device producing more heat transfer according to the various imposed conditions as shape, regime, etc.... It seems that the modified one has more thermal effectiveness while comparing with the classical device.

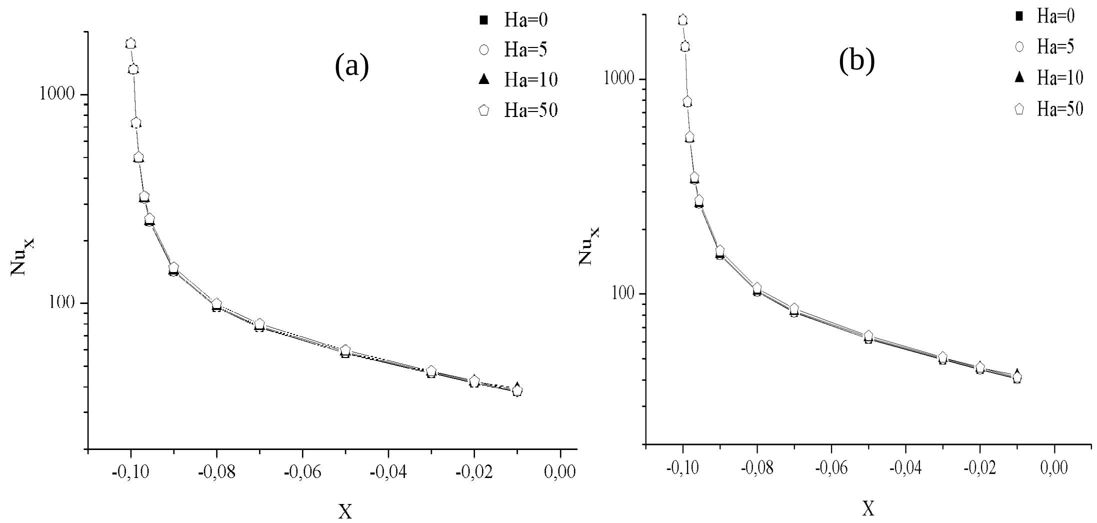


Figure 8: Local Nusselt number Nu_x variation versus the Hartman number for classical system (a), modified system (b)

We notice that the evolutions corresponding to the local Nusselt number Nu_x in both systems has qualitatively the same behaviour with a slight increase in the modified system.

On the other hand, we notice that the pressure field modifies according to the Hartmann number Ha and appears very affected by the high magnetic field values; we note that the pressure force and those of Laplace force interact between them and varies proportionally in both considered systems.

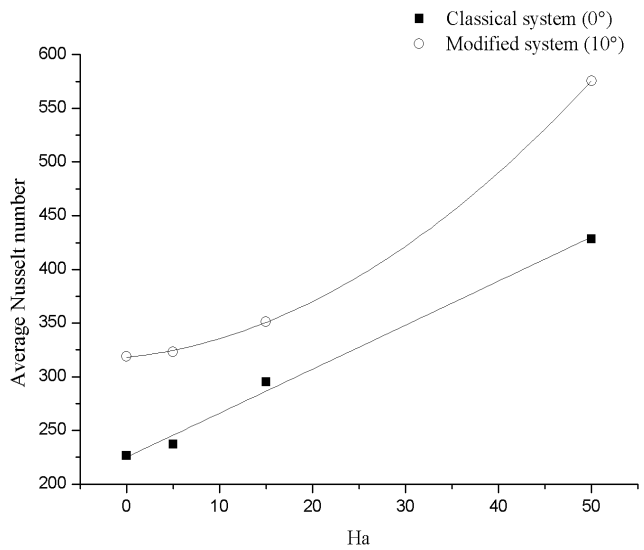


Figure 10: Average Nusselt number $\overline{Nu}$ comparison under the transverse magnetic field effect for Re =2000 in the classical (α=0 °) and (b) modified (α=10 °) systems.

5 Conclusion

We presented numerical results by following a simulation based on parameters dimensionless by means of the software "Fluent". At first, we examined the influence which brings the angular effect on the flow in terms of heat transfer rate and efficiency. Secondly, we were able to deduce the forced convection importance in the thermal properties development of the process, particularly in the modified Graëtz processing system. Afterward, we accentuated that the magnetic field can reduce the intensity of the forced convection in the classical system while the convective transfer increases in the modified one. In the working conditions so that P_r =0.0113 and R_e = 2000, the exchanges are enhanced as the mean Nusselt value can reach $\overline{Nu}$ = 414 if we apply a magnetic field for about Ha = 50 in the modified system. According to the obtained results, we determined the most convenient device in terms of efficiency and heat exchanges, and we were able to take of advantage that the modified geometry presents incomparable qualities even in absence of the magnetic field. The main reason is probably due to a better exchange and control of the entrance region.

In future work, it is important to consider the inclination angle α and the aspect ratio Γ together in order to improve the modified Graëtz processing system.

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