IMPLEMENTATION OF k-k$_L$-ω TURBULENCE MODEL FOR COMPRESSIBLE TRANSITIONAL FLOW INTO OPENFOAM

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Abstract

This paper deals with the results of implementation of k-k$_L$-ω RANS turbulence model for compressible transitional flow into OpenFOAM. This model was firstly proposed by Walters and Leylek (2005) and utilizes the approach of laminar kinetic energy in order to predict transition between laminar and turbulent flows. The capability of laminar/turbulent transition modelling is tested for the basic flat plate test cases and for the VKI turbine cascade. The comparison between new implementation, k-k$_L$-ω for incompressible flow supplied in OpenFOAM and $\gamma - Re_\theta$ model from commercial CFD package FINE/Turbo distributed by NUMECA Int. are shown. The properties of the implementation of k-k$_L$-ω model for compressible flow simulations into OpenFOAM are discussed.

Keywords: Turbulence, CFD, transition, RANS, OpenFOAM

1 Introduction

The laminar/turbulent transition exerts considerable influence on the loss and heat transfer. Therefore the correct transition evaluation is fundamental in many technical applications including the research of compressible flow through turbine cascades. OpenFOAM is the open-source CFD software package which utilizes the finite volume method. Although OpenFOAM includes k-k$_L$-ω model for incompressible flow calculations, this ready-made code gives us wrong results even for basic test cases. Note that the clarification of errors in this code are available in [1]. We implemented k-k$_L$-ω turbulence model for compressible flow in order to build reliable turbulence model for investigation of compressible flow through turbine cascades.

2 Mathematical model

2.1 Navier-Stokes equations

The viscid compressible flow of perfect gas can be described by the set of Favre-averaged Navier-Stokes equations:

$$\frac{\delta\rho}{\delta t} + \frac{\delta(\rho u_i)}{\delta x_j} = 0, \tag{1}$$

$$\frac{\delta(\rho u_i)}{\delta t} + \frac{\delta(\rho u_i u_j)}{\delta x_j} = -\frac{\delta p}{\delta t} + \frac{\delta(t_{ij}+\tau_{ij})}{\delta x_j}, \tag{2}$$

$$\frac{\delta(\rho E)}{\delta t} + \frac{\delta[(\rho E+p)u_j]}{\delta x_j} = \frac{\delta}{\delta x_j}\left[u_i\left(t_{ij} + \tau_{ij}\right) + \left(\frac{\mu}{Pr} + \rho\alpha_0\right)\frac{\delta h}{\delta x_j}\right], \tag{3}$$

where ρE is the specific total energy, $h = E + p - 0.5 \cdot u^2$ is the specific enthalpy, $\tau_{ij} = -\rho u_i' u_j{}'$ is the Reynolds stress tensor and $p \ = \rho r T$ is the equation of state. We assume a Newtonian fluid with constant viscosity μ and shear stress tensor $t_{ij} = 2\mu\left(S_{ij} - 1/3\, S_{kk}\delta_{ij}\right)$, where $S_{ij} = 0.5 \cdot \left(\delta u_j/\delta x_j + \delta u_i/\delta x_i\right)$.

2.2 k-k$_L$-ω turbulence model

The k-k$_L$-ω is the transitional model developed by Walters and Leylek (2005, [2]). This model uses the Boussinesq hypothesis to determine the Reynolds stress tensor. Equations (4), (5) and (6) are solved for the turbulent kinetic energy k_T, the laminar kinetic energy k_L and the specific dissipation rate (turbulent time-scale) ω.

$$\frac{D(\rho k_T)}{Dt} = \rho(P_{kt} + R_{BP} + R_{NAT} - \omega k_t - D_T) + \frac{\delta}{\delta x_j}\left[\left(\mu + \frac{\rho\alpha_T}{\sigma_k}\right)\frac{\delta k_t}{\delta x_j}\right] \tag{4}$$

$$\frac{D(\rho k_L)}{Dt} = \rho(P_{kL} - R_{BP} - R_{NAT} - D_L) + \frac{\delta}{\delta x_j}\left(\mu\frac{\delta k_L}{\delta x_j}\right) \tag{5}$$

$$\frac{D(\rho\omega)}{Dt} =$$

$$\rho\left[C_{\omega 1}\frac{\omega}{k_T}P_{kT} + \left(\frac{C_{\omega R}}{f_W} - 1\right)\frac{\omega}{k_T}(R_{BP} + R_{NAT}) - C_{\omega 2}\omega^2 + \right.$$

$$\left. + C_{\omega 3} f_\omega \alpha_T f_W^2 \frac{\sqrt{k_T}}{d^3}\right] + \frac{\delta}{\delta x_j}\left[\left(\mu + \frac{\rho\alpha_T}{\sigma_\omega}\right)\frac{\delta\omega}{\delta x_j}\right] \tag{6}$$

Terms $\frac{D(\rho k_T)}{Dt}$, $\frac{D(\rho k_L)}{Dt}$ and $\frac{D(\rho\omega)}{Dt}$ represent advection, P_{kt} and P_{kL} represent production, R_{BP} and R_{NAT} represent redistribution rather than production, because they appear with opposite signs in equations (4) and (5). These terms express bypass and natural transition, respectively. They are of the form:

$$R_{BP} = C_R\beta_{RB}k_L\omega/f_W, \tag{7}$$

$$R_{NAT} = C_{R,NAT}\beta_{NAT}k_L\Omega. \tag{8}$$

Description of the other terms as well as model constants can be found in [1].

The bypass transition is governed by the threshold function:

$$\beta_{BP} = 1 - \exp\left(-\frac{\Phi_{BP}}{A_{BP}}\right) \text{ with } \Phi_{BP} = \max\left(\frac{k_T}{\nu\Omega} - C_{BP,crit}, 0\right). \tag{9}$$

The natural transition is governed by the threshold function:

$$\beta_{NAT} = 1 - \exp\left(-\frac{\Phi_{NAT}}{A_{NAT}}\right) \text{ with } \Phi_{NAT} = \max\left(Re_\Omega - \frac{C_{NAT,crit}}{f_{NAT,crit}}, 0\right). \tag{10}$$

The Reynolds number of vorticity $Re_\Omega = d^2\Omega/\nu$ (Ω is the vorticity tensor and d wall distance) grows with the boundary layer thickness. The kinetic laminar energy is the energy of streamwise non-turbulent fluctuations (Tollmien-Schlichting waves) in the pre-transitional region. These fluctuations appear in the laminar boundary layer when Re_Ω reaches the constant value named $C_{TS,crit}$ (see [1]). It starts the production of k_L by the source term P_{kL}. The kinetic laminar energy starts to change into turbulent kinetic energy k_T, when Re_Ω reaches the $C_{NAT,crit}/f_{NAT,crit}$ value. This change initiates the natural transition. The bypass transition is triggered when the ratio $k_T/(\nu\Omega)$ exceeds the $C_{BP,crit}$ value (Eq. 9). Note that $C_{TS,crit}$, $C_{BP,crit}$ as well as $C_{NAT,crit}$ are model constants.

3 Results

3.1 Parallel flow over flat plate

The implementation of k-k$_L$-ω for compressible flow calculations into OpenFOAM (version 2.3.0) was applied for solving 2D transitional flow over flat plate. We suppose test cases T3A and T3A- with zero streamwise pressure gradient [3]. Figure 1 illustrates the computational domain. The region upstream the leading edge consists of 35x105 structured quadrilateral cells and the one above the flat plate of 600x105 quadrilateral cells. The mesh is refined near the wall to ensure $y^+ \leq 1$. The boundary conditions (see Table 1) respect the compressibility.

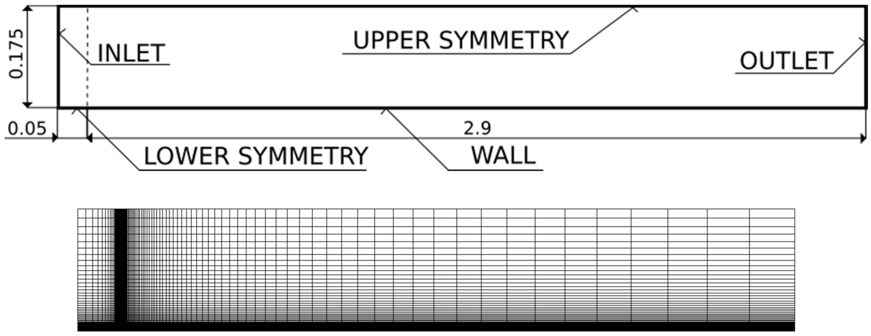

Figure 1: The computational domain of T3A and T3A- test cases.

Table 1: T3A and T3A- boundary conditions. The homogenous Neumann condition is marked ZG. Other boundary conditions were prescribed as symmetry (slip condition).

	U (m/s)	p (kPa)	T (K)	k_T (m^2/s^2)	k_L (m^2/s^2)	ω (s^{-1})
Inlet T3A	5.4	ZG	293.15	0.04763	0	23.8
Inlet T3A-	19.5	ZG	293.15	0.04723	0	23.383
Outlet T3A	ZG	101	ZG	ZG	ZG	ZG
Outlet T3A-	ZG	100.46	ZG	ZG	ZG	ZG
Wall	0	ZG	ZG	0	0	ZG

Both test cases were solved using algorithm SIMPLE. The implemented turbulence model is compared with k-k$_L$-ω model for incompressible flow supplied with OpenFOAM 2.3.0 , k-$\omega\,SST$ model with $\gamma - Re_\Theta$ transition model from commercial CFD package FINE/Turbo from NUMECA Int. and experiment data [3]. Figures 2 and 3 show distribution of the skin friction coefficient. One can see that the k-k$_L$-ω model as well as $\gamma - Re_\Theta$ gives quite good agreement with experiment data. k-k$_L$-ω model shifts transition insignificantly downstream and overshoots the theoretical correlation for fully turbulent flow. This overshoot corresponds to application of turbulence model based on transport equations.

3.2 Flow through VKI turbine cascade

Last test case is the 2D flow through VKI turbine cascade [4, 5]. The blade chord is $c = 300$ mm, the length of the suction side of the blade is $s_{max} = 356$ mm, the pitch to chord ratio is t/c = 0.7 and the stagger angle is $\beta = 49.83°$. The mesh consists of 62049 quadrilateral and triangular cells (see Figure 4). The structured hyperbolic mesh is situated near the blade with $y^+ \leq 1$. The flow with M_{2is} = 0.0884, Re_{2c} = 560000, inlet turbulent intensity Tu = 1.5%, dynamic viscosity μ = 1.2984e-05 and inlet angle α = 0° is considered. Boundary conditions are shown in Table 2. Figure 5 shows distribution of skin friction coefficient $C_f = 2\tau_w / (\rho_e\, U_e^2)$ on the suction side related to free-stream velocity U_e at y = 0.05 m and ρ_e = 1.1885 kg/m^3. The comparison with experiment data [4, 5] shows that k-k$_L$-ω model underestimates friction coefficient. The figure suggests that calculated boundary layer is near to separation, but the position of transition onset is captured well. The transition is driven by limit constants as was mentioned above. Nevertheless it is known, that the transition onset depends, among other things, on pressure gradient of external flow. It can be reason of difference between CFD results and experiment data, because decision constants do not depend on pressure gradient.

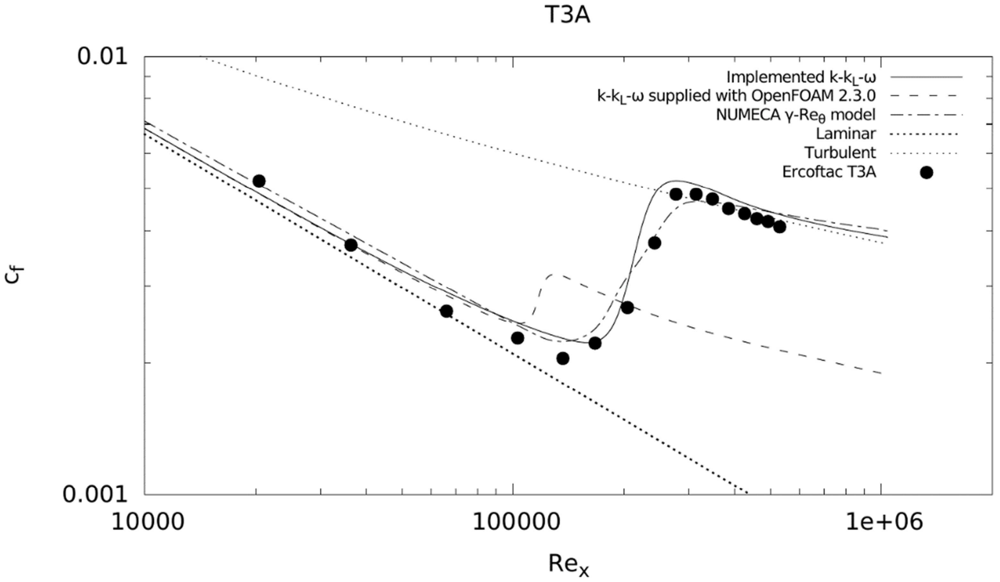

Figure 2: Distribution of the skin friction coefficient for T3A test case.

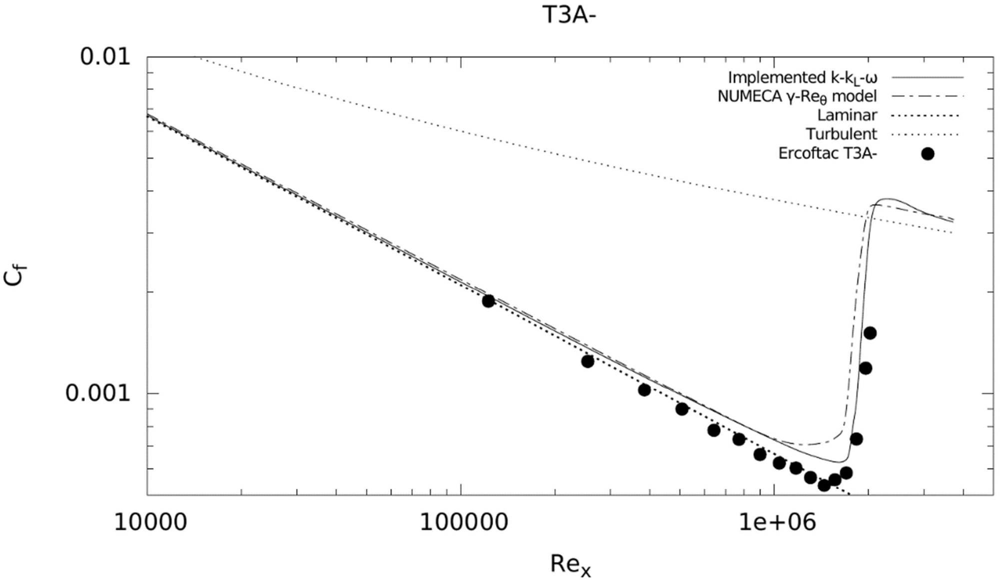

Figure 3: Distribution of the skin friction coefficient for T3A- test case.

Table 2: VKI turbine cascade- boundary conditions. The homogenous Neumann condition is marked ZG. Other boundary conditions are periodic.

	U (m/s)	p (kPa)	T (K)	k_T (m^2/s^2)	k_L (m^2/s^2)	ω (s^{-1})
Inlet	$\alpha = 0°$	$p_{tot} = 100.548$	T_{tot} = 293.7	0.037922	0	16
Outlet	ZG	$p_{stat} = 100$	ZG	ZG	ZG	ZG
Blade	0	ZG	ZG	0	0	ZG

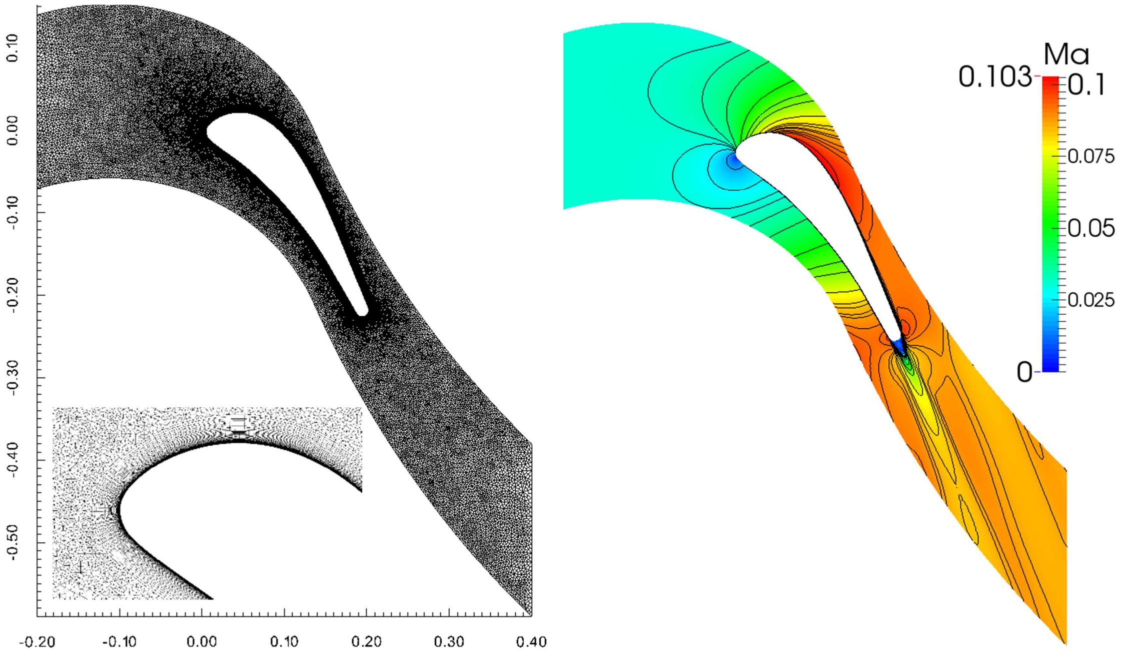

Figure 4: The computational domain of VKI turbine cascade (left) and the Mach number distribution computed using solver rhoSimplecFoam with implemented turbulence model $k - k_L - \omega$ (right).

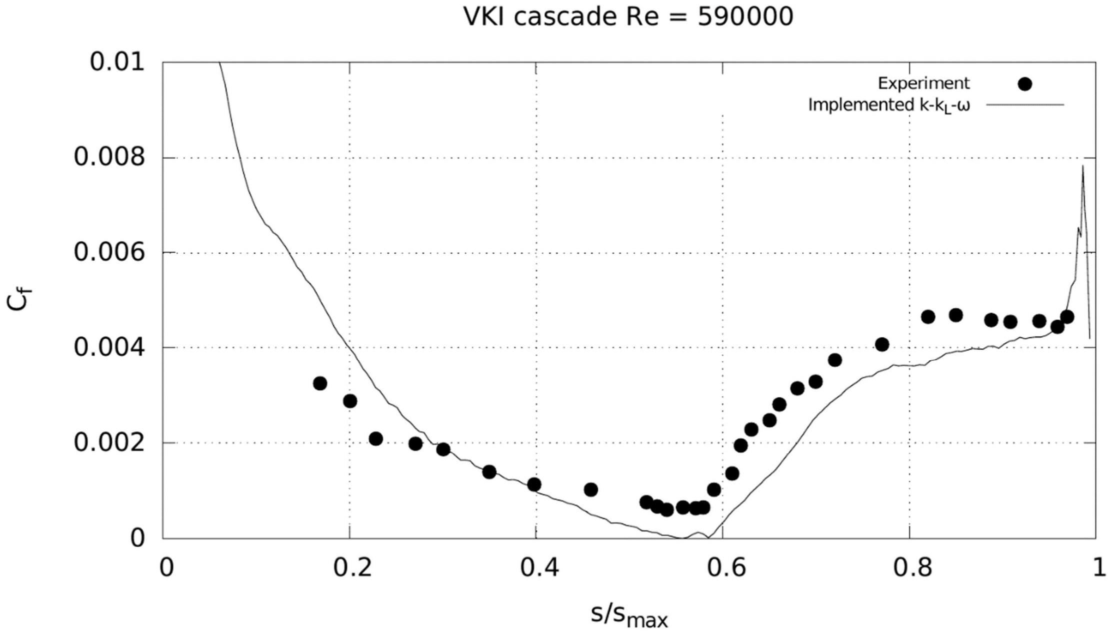

Figure 5: Distribution of the skin friction coefficient along the suction side of the VKI blade.

4 Conclusions

Results of implementation of k-k_L-ω turbulence model for compressible flow into OpenFOAM were presented. Trivial flat plate transitional flows were calculated very well. These results obtained from implemented model are comparable with results from $\gamma - Re_\theta$ model. The start of transition as well as the transition length are even captured in good agreement with ERCOFTAC data. The VKI test case was performed. These results are comparable with experiment data, however the k-k_L-ω model underestimated the skin friction coefficient. The k-k_L-ω utilizes local variables such as Reynolds number of vorticity instead of Reynolds number based on boundary layer thickness. This model is suitable for complex geometries thanks to this fact. On the other hand, we have not found any experimental investigation of pressure gradient dependency of decision parameters. Hence the future development will aim to investigation of the pressure gradient dependency as well as improvement of the turbulent heat transfer model.

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