SPREADING RIVULET WITH DECREASE IN FLOW RATE INDUCED BY DISSIPATION IN UNDERLYING WETTING FILM

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Abstract

Rivulet type flow down an inclined plate is of great importance in many engineering areas including the packed columns design or heat exchangers calculations. In our previous work, we derived a computationally inexpensive method to determine the size of the gas-liquid interface of a rivulet flowing down an inclined wetted plate in the case of negligible gravity effects on the gas-liquid interface shape. In this paper, we propose a generalization of the solution for a non-constant liquid flow rate. Such a generalization provides a way to study the rivulet behavior for different ratios of an underlying film thickness and liquid capillary length.

Keywords: rivulet, fluid dynamics, liquid spreading.

1 Introduction

Even with the ever-growing power of computers, a seemingly simple problem of a gravity driven spreading flow of a liquid is still too complex for parametric studies via CFD algorithms. Hence, there is still a need for simplified solutions to such a problem.

In our previous work, we presented a computationally inexpensive method for calculations of the gas-liquid interface (GLI) size of a gravity driven trickle of a liquid spreading down on (or under) an inclined plate [1]. Later on, based on the work of Duffy and Moffat [2], we generalized the obtained solution for the cases of a rivulet for which the gravity effects on the GLI shape cannot be neglected and we proposed a simple algorithm for approximation of a velocity field in such a spreading rivulet [3]. For the details on the geometry of rivulet spreading and for basic notation, see Fig. 1.

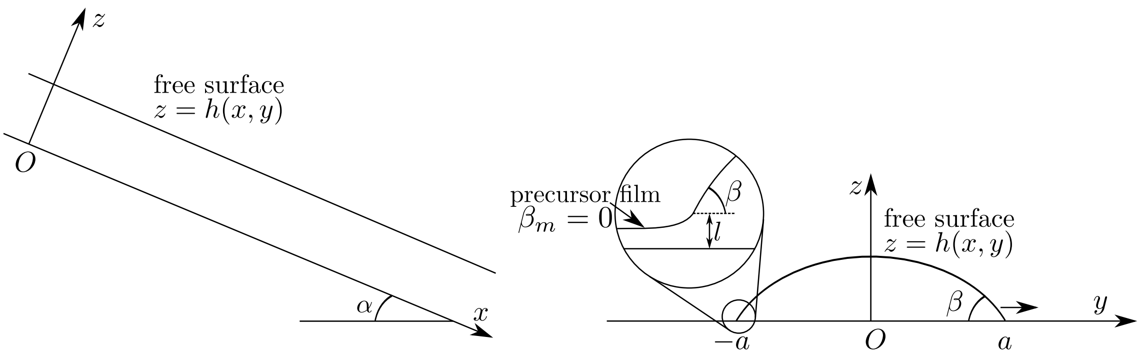

Figure 1: Used coordinate system with the basics of rivulet spreading notation. α is the plate inclination angle, β and β_m are the apparent and the microscopic contact angles, a is the rivulet half width. Letter τ stands for a point where the outer and inner solutions for $h(x,y)$ are stitched together.

Up to now, during the derivation of all the methods and algorithms, we assumed the liquid flow rate to be constant. However, during the flow rate calculation, the liquid in the underlying film is neglected. Thus, an assumption of the constant liquid flow rate is valid only for very small thicknesses of the wetting film. For being able to use the thickness of an underlying liquid layer as a free parameter a more complex model is needed. Such a model has to take into account the loss of a liquid due to its dissipation in the wetting film (see Fig. 2).

In this paper a model coupling the change in the rivulet dynamic contact angle and the change in the liquid flow rate along the plate for the case of negligible gravity effects on GLI shape is derived and non-dimensionalised. A physical interpretation of the obtained results is provided.

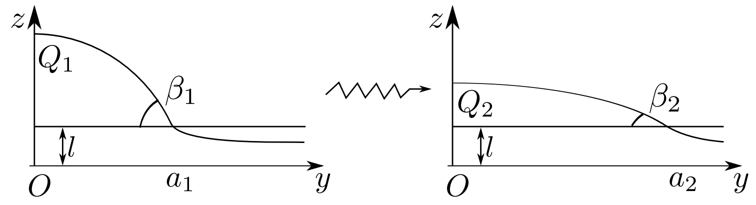

Figure 2: Change in liquid flow rate during spreading. As the rivulet width, a, increases, its contact angle, β, height and volumetric flow rate, Q, decrease. Eventually, for $x \to \infty$ (case of an infinitely long plate), $\beta \to 0$ and $Q \to 0$ and the rivulet completely dissipates in the underlying wetting film.

2 Model derivation

For the sake of brevity of the derivation, we will limit ourselves to study of the rivulet dynamic contact angle and the liquid flow rate in it. An interested reader can find the link between these two quantities and the rivulet GLI shape in [2] and [3].

Change in rivulet dynamic contact angle along the plate: As it was proposed in [1] and [3], with the therein listed simplifying assumptions one can substitute for time, t, in the Cox-Voinov law [4],

$$\beta(t)^3 = 9\frac{\mathrm{d}a(t)}{\mathrm{d}t}\frac{\mu}{\gamma}\ln\left(\frac{a(t)}{2e^2 l}\right), \tag{1}$$

using a transformation derived from speed of movement of τ along the plate [1],

$$t = \varpi x, \qquad \varpi = \frac{2\mu}{\rho g \sin\alpha l^2} \tag{2}$$

Also, it is possible to substitute in (1) for the rivulet width, a, from the relation between the rivulet width, flow rate and dynamic contact angle (a, Q and β, respectively) proposed in [2]

$$a(x) = \left(\frac{4\mu Q(x)}{105\rho g \sin\alpha}\right)^{1/4}\frac{1}{\beta(x)^{3/4}}. \tag{3}$$

This way, one arrives at the following relation for change in β along the plate,

$$\frac{\mathrm{d}\beta}{\mathrm{d}x} = \left[-\frac{4}{27}\frac{\varpi\gamma}{\eta\mu}\frac{\beta^{19/4}}{Q^{1/4}} + \frac{1}{3}\frac{\beta}{Q}\frac{\mathrm{d}Q}{\mathrm{d}x}\right]\ln^{-1}\left(\frac{\eta}{2\mathrm{e}^2 l}\frac{Q^{1/4}}{\beta^{3/4}}\right), \qquad \eta = \left(\frac{105\mu Q}{4\rho g}\right)^{1/4} \tag{4}$$

Change in liquid volumetric flow rate in rivulet along the plate: To be able to solve the equation (4), one needs to couple it with a function defining the liquid volumetric flow rate along the plate. To find such a coupling, we may divide the liquid flow rate between the edges of the

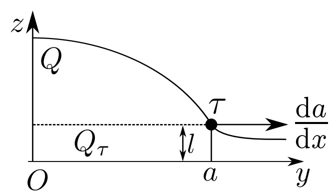

Figure 3: Details of two parts of liquid flow rate in a rivulet spreading down a wetted plate. Part of the liquid flow rate representing the flow rate into the rivulet is denoted as Q (above the dashed line at $z = l$). Volumetric flow rate of liquid in the film underlying the rivulet is denoted as Q_τ.

rivulet in two parts. First part, denoted Q, is the actual flow rate of the liquid in the rivulet. The other part is a consequence of the assumption of a presence of a wetting film.

A film of thickness approximately equal to l is believed to wet the plate on which is the rivulet spreading. Hence, there is also a liquid flow, Q_τ, which corresponds to the liquid present between the rivulet edges, but belonging to the wetting film (for details see Fig. 3).

Flow rate of the liquid between the rivulet edges but appertaining to the wetting film, Q_τ may be obtained by integrating the relation,

$$Q_\tau = 2\int_0^{a(x)}\int_0^{l} \frac{\rho g \sin\alpha}{2\mu} l^2 \,\mathrm{d}z\,\mathrm{d}y = a(x)\frac{\rho g \sin\alpha}{\mu} l^3. \tag{5}$$

The equation (5) may be then used to derive the differential equation for Q,

$$\frac{\mathrm{d}Q}{\mathrm{d}x} = -\frac{\mathrm{d}Q_\tau}{\mathrm{d}x} = -\frac{\rho g \sin\alpha l^3}{\mu}\frac{\mathrm{d}a}{\mathrm{d}x}\,, \tag{6}$$

in which it is possible to substitute for $\mathrm{d}a/\mathrm{d}x$ from the combination of (1) and (2),

$$\frac{\mathrm{d}Q}{\mathrm{d}x} = -\frac{1}{9}\frac{\gamma}{\mu}\frac{\varpi\rho g \sin\alpha l^3}{\mu}\beta^3 \ln^{-1}\left(\frac{\eta}{2\mathrm{e}^2 l}\frac{Q^{1/4}}{\beta^{3/4}}\right). \tag{7}$$

Resulting and dimensionless models: Combining equations (4) and (7), one may obtain the final model. To obtain a dimensionless system we introduce the following dimensionless variables,

$$\tilde{\beta} = \frac{\beta}{\beta_M}, \quad \tilde{a} = l_c = \sqrt{\frac{\gamma}{\rho g}}, \quad \tilde{Q} = \left(\frac{\eta}{l_c}\right)^4 \frac{Q}{\beta_M^3}, \quad \tilde{x} = \frac{x}{L}. \tag{8}$$

In relations (8), β_M follows from the assumption of small contact angles and represents the value where assumption $\tan\beta \doteq \beta$ ceases to hold with the desired accuracy. The rivulet width is scaled (as the gravity effects on the GLI are neglected) using the liquid capillary length, l_c. The scale used for the rivulet flow rate represents a volumetric flow rate in a non-spreading rivulet of constant width, l_c, and contact angle, β_M flowing down a vertical plate. Finally, the plate length coordinate is scaled by the plate length, L.

Substituting from (8) to (4) and (7) and ommitting the tilde, the following dimensionless system is obtained,

$$\frac{\mathrm{d}\beta}{\mathrm{d}x} = \left[\omega_1 \frac{\beta^{19/4}}{Q^{1/4}} + \frac{1}{3}\omega_2 \frac{\beta^4}{Q}\ln^{-1}\left(\omega_3 \frac{Q^{1/4}}{\beta^{3/4}}\right)\right]\ln^{-1}\left(\omega_3\frac{Q^{1/4}}{\beta^{3/4}}\right) \tag{9}$$

$$\frac{\mathrm{d}Q}{\mathrm{d}x} = \omega_2 \beta^3 \ln^{-1}\left(\omega_3 \frac{Q^{1/4}}{\beta^{3/4}}\right) \tag{10}$$

$$\omega_1 = -\frac{8}{27}\frac{RL}{l}\frac{\beta_M^3}{\sin\alpha}, \quad \omega_2 = -\frac{35}{6}\frac{L}{Rl_c}, \quad \omega_3 = \frac{1}{2\mathrm{e}^2}R, \quad R = \frac{l_c}{l},$$

where the variable R stands for the ratio between the liquid capillary length and the underlying film thickness.

3 Results and Discussion

Four different phase portraits of system (9)-(10) are depicted in Fig. 4. It can be seen that for big values of R (thin underlying film in comparison to the rivulet width scale), the decrease in the rivulet flow rate is negligible. With decrease in R, the importance of dissipation of the liquid in the underlying film increases and for $R < 300$, a region where the liquid dissipation is the most important process appears.

4 Conclusion

Building on our previous work, we were able to derive a model describing the change in a spreading rivulet contact angle coupled with the change in its volumetric flow rate. We assumed the change

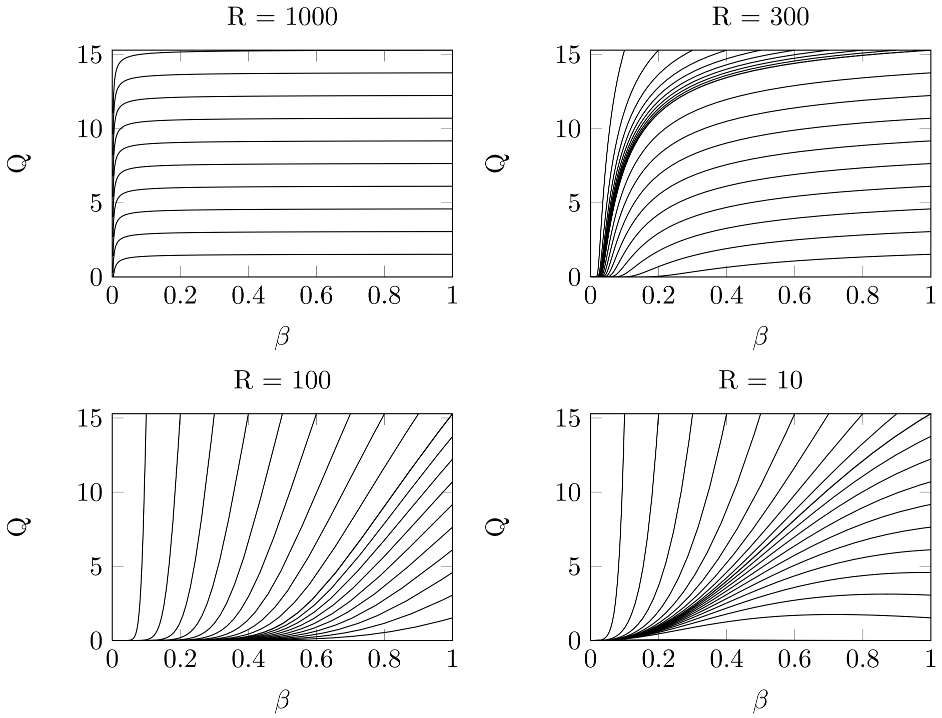

Figure 4: Trajectories of the system (9)-(10) plotted for different values of ratio between the liquid capillary length and the underlying film thickness, R. The used liquid was a silicon oil of $\mu \doteq 10^{-2}\,\mathrm{Pa\,s}$, $\gamma \doteq 10^{-2}\,\mathrm{N\,m^{-1}}$ and $\rho \doteq 10^{3}\,\mathrm{kg\,m^{-3}}$. Plate inclination angle was kept at $\alpha = \pi/4$. Dimensional liquid flow rate was in orders of $1\,\mathrm{ml\,s^{-1}}$ and $\beta_M = 10^{-2}$.

in the flow rate to be induced by the rivulet dissipation in the underlying wetting film. Using the presented model, one may deduce that for ratio between the rivulet capillary length and the film thickness bigger than approximately 300, our previously derived rivulet GLI simulation methods neglecting the change in liquid flow rate are very well usable. Currently we are working on validation of the presented work using CFD simulations.

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Acknowledgments: The presented work was supported by IGA of UCT Prague, under the grant number A2_FCHI_2014_001.