DYNAMIC MODE DECOMPOSITION OF SYNTHETIC JET FLOW AT RE = 329 AND S = 19.7

T. Hyhlík [1]

[1] **Department of Fluid Dynamics and Thermodynamics, Faculty of Mechanical Engineering, Czech Technical University in Prague, Technická 4, 166 07 Prague 6, Czech Republic**

Abstract

This work uses dynamic mode decomposition (DMD) to analyze numerical simulation of axisymmetric vortex street like structure with traveling vortex ring which is connected with synthetic jet creation. The regime of synthetic jet with Reynolds number $Re = 329$ and Stokes number $S = 19.7$ is studied. The analysis is based on the numerical simulation of axisymmetric unsteady laminar flow obtained using ANSYS Fluent CFD code. The theory behind DMD is partly introduced. The structure of DMD modes corresponding to quasi-periodic vortex street is discussed.

Keywords: synthetic jet, modal decomposition, dynamic mode decomposition.

1 Introduction

Existence of synthetic jet is closely related to vortex ring creation. The possibility to generate (synthetize) jet is connected with vorticity flux in the blowing part of period. If there is enough vorticity flux then vortex ring can be observed in the flow field. The created vortex ring is inducing velocity in the direction outside from the orifice. Synthetic jet can be generated only in the case where vortex ring induced velocity is able to overcome suction velocity. Vortex street like structure with traveling vortex ring is typically connected with synthetic jet creation. The dynamics of vortex street can be studied by using modal decomposition technique.

The most commonly used governing parameters of synthetic jet are based on so called slug model, see e.g. [1, 2]. “Slug velocity profile” model introduces dimensionless stroke length and Reynolds number based on some velocity scale as is shown in next section. Velocity scale based on slug model does not necessarily correspond to an appropriate value. It is possible to show that circulation of vortex ring is proportional to Reynolds number based on dimensionless stroke length. As mentioned in the previous paragraph, the vortex ring induced velocity is crucial for the existence of synthetic jet. It is an open question to use some parameter which can be deduced from spatial structure of dynamical modes obtained using DMD.

2 Parameters of synthetic jet

Velocity scale is typically defined as time averaged blowing orifice centerline velocity over an entire cycle

$$U_0 = \frac{1}{T}\int_0^{T_E} u_0(r=0,t)\ \mathrm{d}t, \tag{1}$$

where T_E is blowing time and $T = 1/f$ is oscillation period. Alternatively, velocity scale can be defined as time and spatial averaged blowing orifice velocity

$$\overline{U} = \frac{1}{T_E}\int_0^{T_E}\left(\frac{1}{A}\int_0^{D/2} 2\pi r u_0(r,t)\mathrm{d}r\right)\mathrm{d}t, \tag{2}$$

where D is orifice diameter and A is orifice area. It is possible to show that two previously mentioned velocities are related as

$$\overline{U} = 2U_0. \tag{3}$$

Reynolds number can be defined using time averaged blowing orifice centerline velocity U_0 as

$$Re = \frac{2U_0 D}{\nu} = \frac{\overline{U}D}{\nu}. \tag{4}$$

Stokes number is defined as

$$S = D\sqrt{\frac{2\pi f}{\nu}}, \tag{5}$$

where f is frequency. Stroke length of synthetic jet is length of fluid column that is pushed out during one cycle and can be calculated as $L_0 = U_0 T$. Dimensionless stroke length is defined as

$$L = \frac{L_0}{D} = \frac{Re}{S^2}\pi. \tag{6}$$

3 Dynamic mode decomposition

Results of numerical simulation of flow field does not give understanding of the dynamics in it without additional analyses. Modal decomposition can be used to decompose flow field to the modes with relatively simple spatial structure which revealing behavior of studied flow field. Dynamic mode decomposition was developed based on Koopman analysis of dynamical system [3]. DMD algorithm approximates Koopman modes from finite set of data. This section is mainly based on the reference [4]. Let the velocity field be represented by ensemble of snapshots sampling at interval Δt with a form of matrix $\mathbf{V}_1^N = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_N\}$, where the column vector $\mathbf{v}_j$ contains velocity field at jth time step. Let's consider linear mapping $\mathbf{A}$ that maps each snapshot to the next one in the following way

$$\mathbf{v}_{j+1} = \mathbf{A}\mathbf{v}_j. \tag{7}$$

This mapping is assumed to be same over the full sampling interval $[0, (N-1)\Delta t]$. If the flow fields stem from a nonlinear process, this assumption corresponds to linear tangent approximation [4]. The assumption of a constant mapping between the snapshots $\mathbf{v}_j$ allow us to formulate the sequence of flow fields as a Krylov sequence

$$\mathbf{V}_1^N = \left\{\mathbf{v}_1, \mathbf{A}\mathbf{v}_1, \mathbf{A}^2\mathbf{v}_1, \ldots, \mathbf{A}^{N-1}\mathbf{v}_1\right\}. \tag{8}$$

The goal of DMD is the extraction of dynamic characteristics such as eigenvalues and eigenvectors which are described by linear operator $\mathbf{A}$ based on sequence $\mathbf{V}_1^N$. Suppose that it is possible to write last snapshot $\mathbf{v}_N$ as linear combination of previous $N-1$ snapshots, for sufficiently long sequence of the snapshots

$$\mathbf{v}_N = a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + \ldots + a_{N-1}\mathbf{v}_{N-1} + \mathbf{r}, \tag{9}$$

where $\mathbf{a}^T = \{a_1, a_2, \ldots, a_{N-1}\}$ and $\mathbf{r}$ is residual vector. It is possible to write following relations

$$\mathbf{A}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \ldots, \mathbf{v}_{N-1}\} = \{\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \ldots, \mathbf{v}_N\} = \{\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \ldots, \mathbf{V}_1^{N-1}\mathbf{a}\} + \mathbf{r}\mathbf{e}_{N-1}^T, \tag{10}$$

or in matrix form

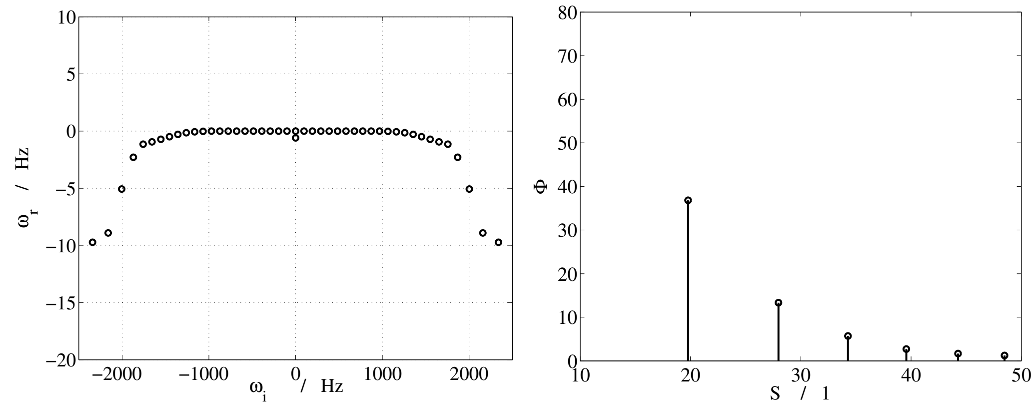
$$\mathbf{A}\mathbf{V}_1^{N-1} = \mathbf{V}_2^N = \mathbf{V}_1^{N-1}\mathbf{S} + \mathbf{r}\mathbf{e}_{N-1}^T, \tag{11}$$

where $\mathbf{e}_{N-1}^T$ is unit vector of dimension $N-1$. Matrix $\mathbf{S}$ is companion matrix

$$\mathbf{S} = \begin{pmatrix} 0 & & & & a_1 \\ 1 & 0 & & & a_2 \\ & \ddots & \ddots & & \vdots \\ & & 1 & 0 & a_{N-2} \\ & & & 1 & a_{N-1} \end{pmatrix}. \tag{12}$$

The eigenvalues of $\mathbf{S}$ are approximations of eigenvalues of linear operator $\mathbf{A}$. Decomposition based on companion matrix $\mathbf{S}$ is mathematically correct, but more robust algorithm based on similar matrix $\tilde{\mathbf{S}}$ is used in this work. Robustness is achieved by preprocessing step using singular value decomposition of data sequence [4] $\mathbf{V}_1^{N-1} = \mathbf{U}\mathbf{\Sigma}\mathbf{W}^H$. Substituting singular value decomposition into (11) and rearranging the resulting expression we have

$$\mathbf{U}^H\mathbf{A}\mathbf{U} = \mathbf{U}^H\mathbf{V}_2^N\mathbf{W}\mathbf{\Sigma}^{-1} = \tilde{\mathbf{S}}. \tag{13}$$



(a) DMD spectrum (b) Norm of spatial part of DMD modes

Figure 1: Dynamic mode decomposition spectrum and norm of spatial part versus Stokes number

Dynamic modes can be calculated using equation [4]

$$\mathbf{\Phi}_i = \mathbf{U}\mathbf{y_i}, \tag{14}$$

where $\mathbf{y}_i$ is ith eigenvector of $\tilde{\mathbf{S}}$

$$\tilde{\mathbf{S}}\mathbf{y}_i = \mu_i \mathbf{y}_i. \tag{15}$$

Eigenvalues μ_i approximate eigenvalues of full mapping matrix $\mathbf{A}$ and provide temporal dynamics of flow field.

4 Numerical simulation

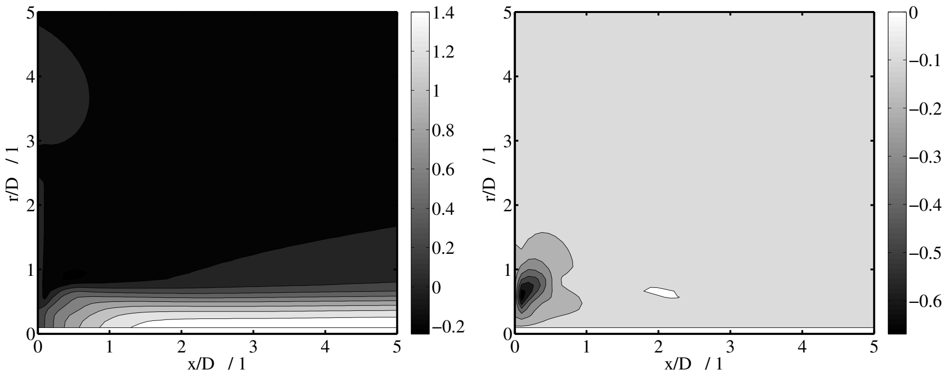
Unsteady incompressible laminar flow simulation has been performed using commercial solver ANSYS Fluent. The flow is assumed to be axisymmetric and is simulated on computational mesh with 81550 cells by using non iterative time advancement method with second order implicit scheme. Fractional step scheme is used for pressure velocity coupling. Convective terms are discretized using third order MUSCL scheme. Both the flow in the orifice and actuator cavity are included in the simulation to get more accurate results. The effect of oscillating diaphragm is replaced by velocity boundary condition which should guarantee achieving of required Reynolds number [5]. More details about numerical simulation is in the reference [5].

5 Dynamic mode decomposition of flow field

Analyzed data are normalized by using averaged blowing orifice centerline velocity over an entire cycle U_0. Obtained DMD spectrum is depicted in the figure 1(a). It is visible that most modes are quasi-neutral stable with $\omega_r = 0$ which is reasonable for the case of quasi periodic flow. There is not growing mode in the DMD spectrum with positive ω_r and only few dumped modes with negative ω_r. Norm of spatial part of DMD mode versus mode Stokes number is shown in the figure 1(b). The first dynamic mode corresponds with $S = 19.7$, the second mode corresponds with double frequency and $S = 27.8$, the third mode has triple frequency and $S = 34.1$ and so on.

The spatial pattern of mode with zero frequency is in the figure 2. This mode has only real part and is very close to time averaged flow field. The behavior in the figures 2 and 3 is confirming existence of synthetic jet in this case. There is a flow around the wall in the radial direction and jet flow is in the axial direction.

Spatial pattern of first four dynamic modes is depicted in the figure 5. Both real parts and imaginary parts of modes are shown in the figure 5. It is possible to observe finer and finer spatial structure of dynamic mode with increasing Stokes number of mode. There is a space shift in axial



(a) Axial velocity (b) Radial velocity

Figure 2: Spatial pattern of zero mode (real part)

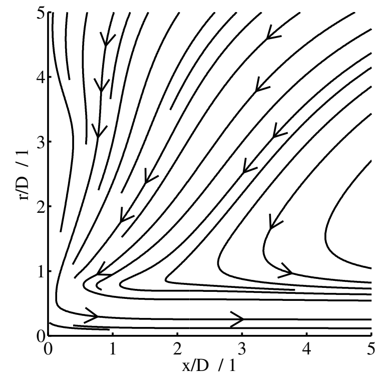
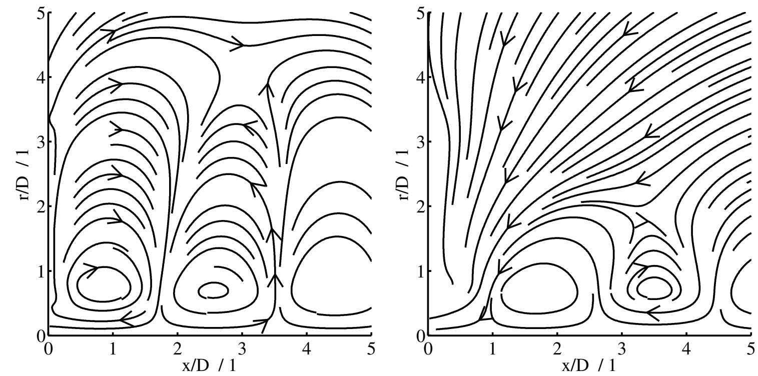
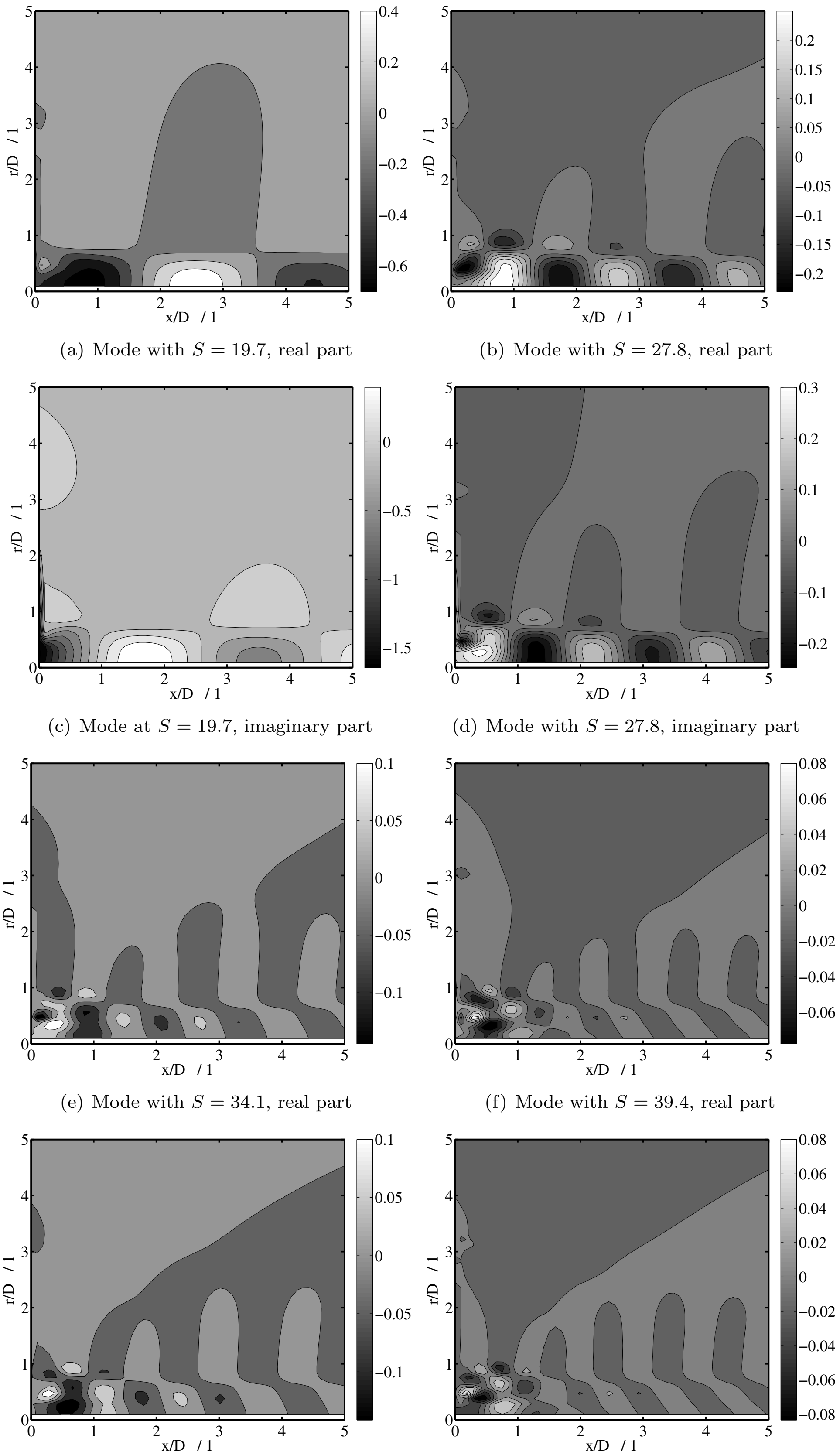


Figure 3: Vector lines of zero mode



(a) real (b) imag

Figure 4: Vector lines of first mode



(a) Mode with $S = 19.7$, real part

(b) Mode with $S = 27.8$, real part

(c) Mode at $S = 19.7$, imaginary part

(d) Mode with $S = 27.8$, imaginary part

(e) Mode with $S = 34.1$, real part

(f) Mode with $S = 39.4$, real part

(g) Mode with $S = 34.1$, imaginary part

(h) Mode with $S = 39.4$, imaginary part

Figure 5: Axial velocities of 1, 2, 3 and 4 modes

direction between real and imaginary part of each dynamic mode. The topology of dynamic modes is characterized by alternation of local minimums and maximums in the field of axial velocity in the axial direction. The absolute values of local minimums and maximums are decreasing with increasing mode number.

The alternation of local minimums and maximums in axial velocity indicates vortex structure depicted in the figure 4. There is a space shift of vortex structures shown in the figure 4 in axial direction between real and imaginary part of dynamic mode. Figure 4 shows also that vortex distance is not multiple of integer of dimensionless stroke length $L = 0.8477$ which is based on simple slug model. Last but not least it is possible to observe opposite vortex rotation in the real and in the imaginary part of first dynamic mode. This behavior was also observed in the case of proper orthogonal decomposition (POD) between two real POD modes [6] in the case of vortex street.

6 Conclusions

Dynamic mode decomposition of synthetic jet flow at $Re = 329$ and $S = 19.7$ has been done. The topology of first four dynamic modes is shown together with zero mode corresponding with time averaged flow field. It has been shown that the first dynamic mode corresponds with basic frequency related to Stokes number $S = 19.7$ and other modes corresponds with multiple of integer of basic frequency. The space shift between real and imaginary part of dynamic mode is observed similarly like between two POD modes [6]. Vortex distance in the dynamic modes is not multiple of integer of dimensionless stroke length L based on slug model. It has been proven that dynamic mode decomposition can be used to study synthetic jet flow and to identify its characteristics.

Acknowledgment

The support from *Czech Science Foundation* under *Grant No. 14-08888S* is gratefully acknowledged.

References

[1] R. Holman, Y. Utturkar, R. Mittal, B. L. Smith, L. Cattafesta, Formation Criterion for Synthetic Jet, *AIAA Journal*, **43(10)**, (2005), pp. 2110-2116

[2] Z. Trávníček, Z. Broučková, J. Kordík, Formation Criterion for Axisymmetric Synthetic Jets at High Stokes Numbers, *AIAA Journal*, **50(9)**, (2012), pp. 2012-2017

[3] C. W. Rowley, I. Mezić, S. Bagheri, P. Schlatter, D. S. Henningson, Spectral Analysis of Nonlinear Flows, *J. Fluid Mech.*, **641**, (2009), pp. 115-127

[4] P. J. Schmidt, Dynamic Mode Decomposition of Numerical and Experimental Data, *J. Fluid Mech.*, **656**, (2010), pp. 5-28

[5] T. Hyhlík, H. Netřebská, Numerical Simulation of Axisymmetric Synthetic Jet Flow Using ANSYS Fluent Solver, In: *33th Conference of Departments of Fluid Mechanics and Thermomechanics*, Eds.: P. Šafařík, H. Netřebská, J. Nožička, Prague, Sept. 9-11, 2014, pp. 21-25

[6] T. Hyhlík, P. Železný, J. Čížek, Visualization and Modal Decomposition of Vortex Street Behind Circular Cylinder, *EJP Web of Conferences*, **45**, (2013), 01043