TOWARDS PRESSURE GRADIENT SENSITIVE TRANSITIONAL $k-k_L-\omega$ MODEL: THE NATURAL TRANSITION FOR LOW RE AIRFOILS

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Abstract

The work deals with the development of a pressure gradient sensitive variant of a three-equation RANS model for turbulent and transitional flows. The model is based on the laminar kinetic energy approach by Walters and Cokljat [1]. The revisited model [2] was successfully validated for zero or mild pressure gradient flows with the so-called bypass transition (see e.g. [2]). In the case of natural transition in adverse pressure gradient flows the model usually tends to predict the transition too late, see [3], [4], [5]. The aim of present work is to improve the model for the case of flows with very low free-stream turbulence levels with adverse pressure gradient. As a first step we focus the attention to the onset of linear instability.

Keywords: turbulence, laminar-turbulent transition, laminar kinetic energy.

1 Introduction

The main goal of this article is to improve the three-equation model proposed by Walters and Cokljat [1] for the flows at very low free-stream turbulence levels subjected to adverse pressure gradient. The motion of the incompressible flow is described by the system of Reynolds averaged Navier-Stokes equations with Reynolds stresses approximated by the eddy viscosity model. The eddy viscosity is obtained from the three-equation $k-k_L-\omega$ model.

1.1 Three-equation $k-k_L-\omega$ model

The three equation model of [1] is assumed with the transport equations for the turbulent kinetic energy k_T, the laminar kinetic energy k_L, and the specific dissipation rate ω. The equations are

$$\begin{aligned}
\frac{Dk_T}{Dt} &= P_{k_T} + R_{BP} + R_{NAT} - \omega k_T - D_T + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\alpha_T}{\sigma_k}\right)\frac{\partial k_T}{\partial x_j}\right], \\
\frac{Dk_L}{Dt} &= P_{k_L} - R_{BP} - R_{NAT} - D_L + \frac{\partial}{\partial x_j}\left[\nu\frac{\partial k_L}{\partial x_j}\right], \\
\frac{D\omega}{Dt} &= C_{\omega 1}\frac{\omega}{k_T}P_{k_T} + \left(\frac{C_{\omega R}}{f_W} - 1\right)\frac{\omega}{k_T}(R_{BP} + R_{NAT}) - C_{\omega 2}\omega^2 \\
&\quad + C_{\omega 3} f_\omega \alpha_T f_W^2 \frac{\sqrt{k_T}}{d^3} + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\alpha_T}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right].
\end{aligned} \tag{1}$$

We will describe here only key terms responsible for natural transition and we refer interested reader to [2] for full description of the model.

1.2 Natural transition model

The natural transition mode is driven by the R_{NAT} term in the previous equation. R_{NAT} is given as

$$R_{NAT} = C_{R,NAT}\beta_{NAT}k_L\Omega, \tag{2}$$

$$\beta_{NAT} = 1 - \exp\left(-\frac{\phi_{NAT}}{A_{NAT}}\right), \tag{3}$$

$$\phi_{NAT} = \max\left(Re_\Omega - \frac{C_{NAT,crit}}{f_{NAT,crit}}, 0\right), \tag{4}$$

$$f_{NAT,crit} = 1 - \exp\left(-C_{NC}\frac{\sqrt{k_L}d}{\nu}\right). \tag{5}$$

The production of laminar kinetic energy k_L is $P_{k_L} = \nu_{T,l}S^2$ with

$$\nu_{T,l} = \min\left\{f_{\tau,l}C_{l1}\frac{\Omega\lambda_{eff}^2}{\nu}\sqrt{k_{T,l}}\lambda_{eff} + \beta_{TS}C_{l2}Re_\Omega d^2\Omega, \frac{k_L + k_{T,l}}{2S}\right\}, \tag{6}$$

where

$$Re_\Omega = \frac{d^2\Omega}{\nu}, \tag{7}$$

$$\beta_{TS} = 1 - \exp\left(-\frac{\max(Re_\Omega - C_{TS,crit}, 0)^2}{A_{TS}}\right). \tag{8}$$

It can be seen from the previous formulas that the onset of the natural transition is governed by the threshold function β_{NAT}. It changes its value from zero to one as soon as $Re_\Omega > C_{NAT,crit}/f_{NAT,crit}$, see eq. (4). Since $f_{NAT,crit}$ is increasing with k_L, the onset of natural transition depends on the appropriate value of k_L. The production of k_L itself is given by P_{K_L} and hence by $\nu_{T,l}$, see (6). We focus the attention to flows at very low free-stream turbulence levels, therefore we assume that the laminar fluctuations are mainly due to the so called Tollmien-Schlichting waves[6] and that the second term containing β_{TS} in the eq. (6) plays the most important role.

The threshold function β_{TS}, see eq. (8), activates the production of laminar fluctuations as soon as $Re_\Omega > C_{TS,crit}$ where the model parameter $C_{TS,crit}$ equals to 1000 in the original model proposed by Walters and Cokljat.

2 The onset of Tollmien-Schlichting waves

2.1 Zero pressure-gradient flows

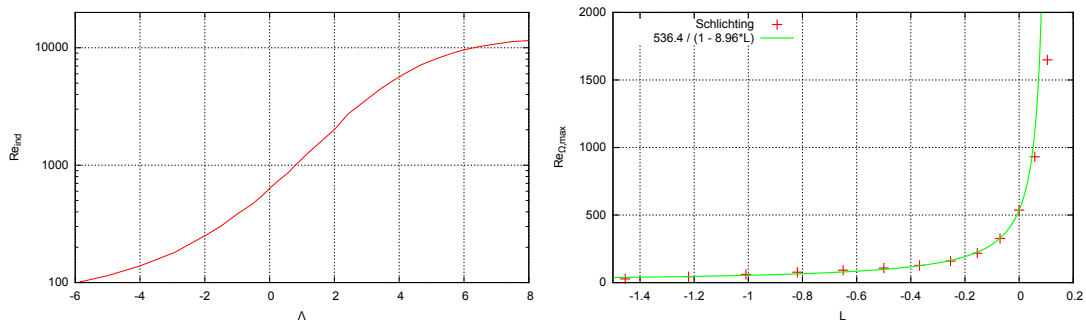
The appropriate value of k_L at the transition point depends on the position where the production of k_L starts and on the right growth rate of k_L. We focus our attention here to the start of production. The start of P_{k_L} is characterized by $Re_\Omega > C_{TS,crit}$ in the original model. Contrary to the original model we assume here that the Tollmien-Schlichting waves appear as soon as the flow reaches the limit of linear stability given by the Orr-Sommerfeld equation [6]. The limit of stability for the flat plate flows at zero pressure gradient is

$$Re_{ind} = \left(\frac{U_\infty\delta^*}{\nu}\right)_{ind} = 520, \tag{9}$$

where U_∞ is the free-stream velocity, δ^* is the displacement boundary layer thickness, and Re_{ind} is the critical Reynolds number. Using the Blasius velocity profile one can devise following relations $Re_{\Omega,max} = 2.185Re_\theta$ and $\delta^* = 2.591\theta$ one obtains

$$C_{NAT,crit}^{ZPG} = Re_{\Omega,max} = 2.185Re_\theta = 2.185\frac{\theta}{\delta^*}Re_{ind} = 439, \tag{10}$$

which is rather different from the value proposed in the original model.



(a) Re_{ind} as a function of Λ according to [6]

(b) $Re_{\Omega,max}$ as a function of L

Figure 1: Neutral stability limit for the flows with pressure gradient.

2.2 Adverse pressure-gradient flows

In order to make the model sensitive to the pressure gradient we propose to develop a correlation for $C_{NAT,crit}$. We assume the Pohlhausen velocity profile [6]

$$\frac{U(y)}{U_e} = 2\eta - 2\eta^3 + \eta^4 + \frac{\Lambda}{6}\eta(1-\eta)^3, \tag{11}$$

with $\eta = y/\delta$ where δ is the boundary layer thickness, U_e is the velocity at the boundary layer edge, and Λ is the Pohlhausen parameter defined by the following equation

$$\Lambda = \frac{\delta^2}{\nu}\frac{dU_e}{dx}. \tag{12}$$

The stability calculation for this family of velocity profiles was carried out by Schlichting and Ulrich [7]. Figure 1a shows the limit of neutral stability for different values of Λ. Using the analytical form of velocity profiles we can transform the results of Schlichting and Ulrich [7] to a form which is more appropriate for the current model, i.e. we will express the limit of stability in terms of $Re_{\Omega,max}$ as a function of the dimensionless pressure gradient

$$L = Re^2_{\Omega,max}\frac{\nu}{U_e^2}\frac{dU_e}{dx}. \tag{13}$$

The figure 1b shows the data obtained with the transformation of result of Schlichting and Ulrich [7]. Points correspond to the original data and the line is the new correlation for stability limit expressed in terms of Re_Ω:

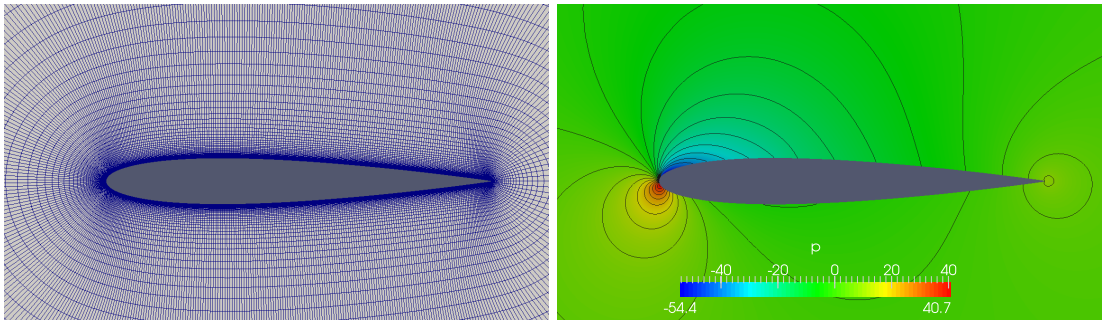
$$C^{APG}_{TS,crit} = \frac{536.4}{1 - 8.963L}, \text{ for } -1.5 \le L \le 0. \tag{14}$$

2.3 Implementation of the new correlation into the modified $k - k_L - \omega$ model

The implementation of the new correlation (14) is relatively straightforward:

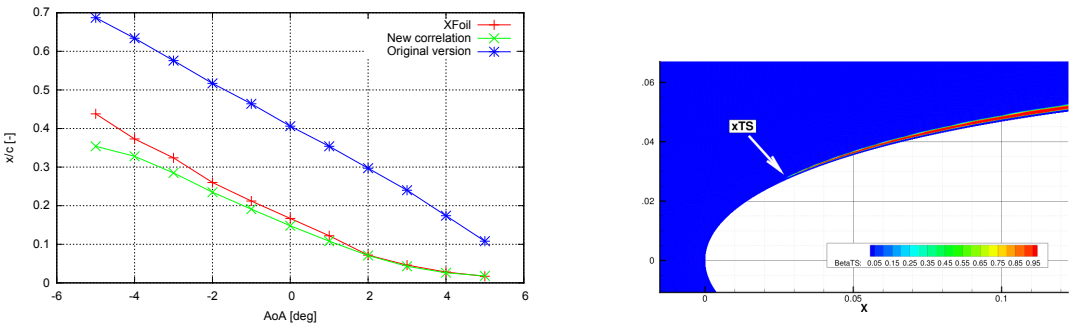
- First of all we calculate the value of the total pressure as $p_{tot} = \max(p + 1/2||\vec{U}||^2)$ where the maximum is taken over whole domain and $\vec{u}$ is the local velocity vector.
- Then we express the value of "external" velocity as $U_e = \sqrt{2(p_{tot} - p)}$ and we calculate $\tilde{L}$ as

$$\tilde{L} = Re^2_\Omega\frac{\nu}{U_e^2||\vec{U}||}\text{grad}U_e \cdot \vec{U}. \tag{15}$$



(a) Part of the mesh (b) Pressure distribution

Figure 2: Mesh and the distribution of the pressure for the flow around NACA 0012 profile at AoA=4°.



(a) Dependence of the onset on α (b) Distribution of β_{TS} for $\alpha = 4°$

Figure 3: Position of Tollmien-Schlichting instability onset

- We replace constant value of $C_{TS,crit}$ by

$$C_{TS,crit} = \frac{536.4}{1 - 8.963 \max(\min(\tilde{L}, 0), -1.5)}. \tag{16}$$

The proposed method requires flow field with constant (or almost constant) value of total pressure p_{tot} in order to be able to calculate local values of U_e.

3 Numerical experiment with NACA 0012 profile

The proposed modification of the $C_{TS,crit}$ was tested for the case of incompressible flows around the NACA 0012 profile with Reynolds number $Re = 6 \times 10^5$ with the free-stream turbulence level $Tu = 0.3\%$ at different angles of attack. The simulation was performed using OpenFOAM 2.3 package coupled with our own implementation of the $k - k_L - \omega$ model using hyperbolic O-type mesh with 500 cells around the profile with first cell size about $y^+ \approx 0.2$. The 2D domain with diameter about 30 chords contained approximately 50 000 cells in total.

In order to test the proposed correlation we carried out several calculations with angles of attack $\alpha = -5° to 5°$ and we evaluated values of x_{TS} defined as the x-coordinate where the β_{TS} becomes non-zero, see eq. (8) and the figure 2b. Calculated values of x_{TS} for the original model and the model including the new correlation (16) are compared to the data obtained with XFoil [8], see fig. 2a. In the later case we plot the position where the amplification factor N becomes non-zero. One can see that the correlation (16) in general moves the onset of the Tollmien-Schlichting instability toward the inlet edge and improves the agreement with XFoil.

However the new correlation improves the prediction of instability position, it has very small effect on the transition itself. On the other hand we have shown in [4] that the natural transition

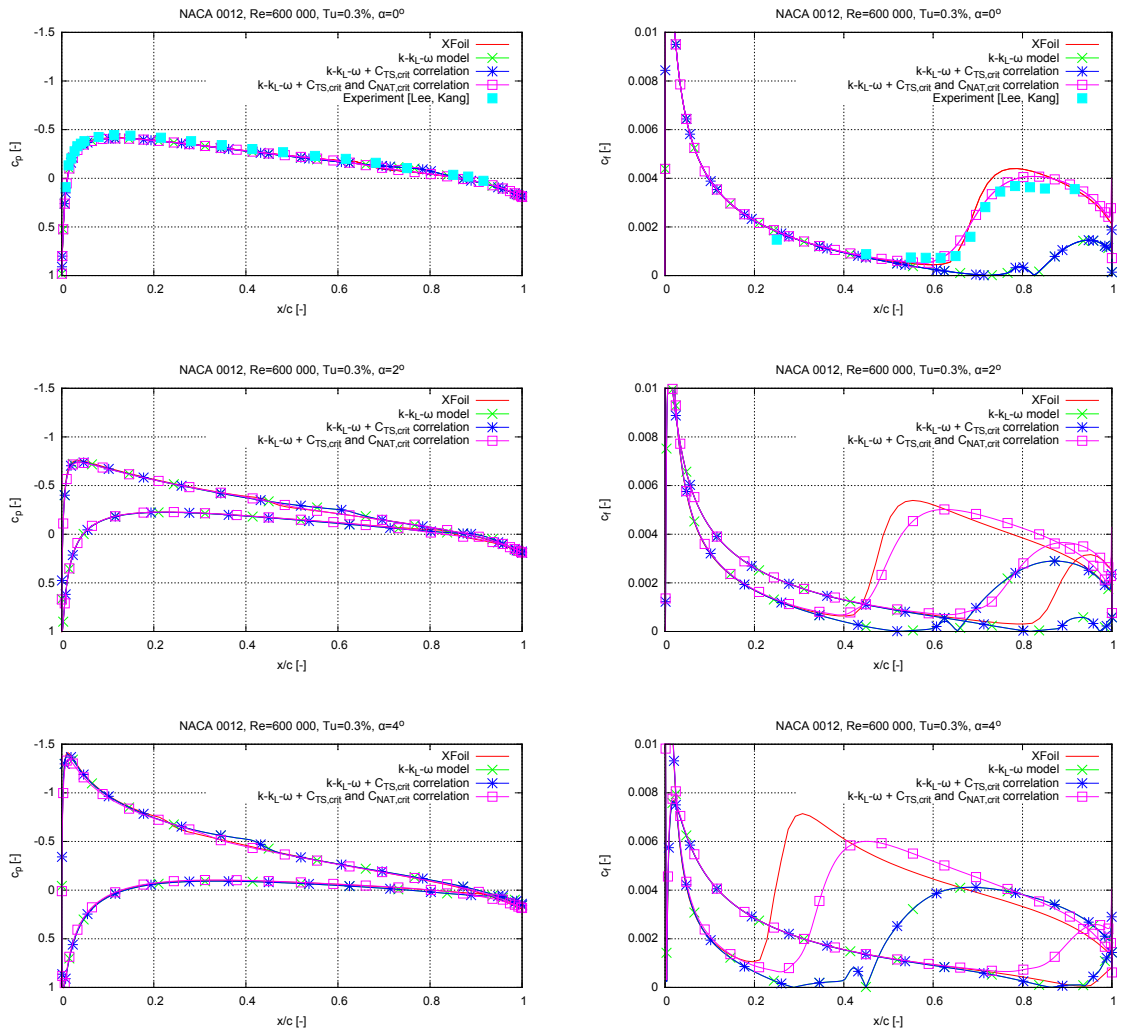


Figure 4: Pressure a friction coefficient calculated with and without correlations for $C_{TS,crit}$ and $C_{NAT,crit}$ and comparison with XFoil.

can be efficiently tuned using $C_{NAT,crit}$ parameter. Therefore we propose to use similar form of pressure gradient dependency also for $C_{NAT,crit}$. As a first attempt, we assume constant ratio $C_{TS,crit}/C_{NAT,crit}$, so the $C_{NAT,crit}$ is

$$C_{NAT,crit} = \frac{1250}{1 - 8.963\max(\min(\tilde{L}, 0), -1.5)}. \tag{17}$$

Figure 4 shows the distribution of the pressure coefficient $c_p = 2(p - p_\infty)/(\rho_\infty u_\infty^2)$ and of the friction coefficient $c_f = 2\tau_w/(\rho_\infty u_\infty^2)$ for angles of attack AoA=0°, 2° and 4°. The different lines represent the data obtained with the original version of the $k - k_L - \omega$ model by [1], the model with non-constant $C_{TS,crit}$ given by (16), and with the model with both non-constant $C_{TS,crit}$ and $C_{NAT,crit}$ given by (16) and (17). The data are compared to reference solution obtained with XFoil and for the case of $\alpha = 0°$ also to experimental data by Lee and Kang [9]. One can see that the non-constant $C_{TS,crit}$ has negligible effect on the transition onset. On the other hand the data obtained with non-constant $C_{NAT,crit}$ correspond very well both to experiment and to the reference data.

4 Conclusion

The new pressure-gradient sensitive three-equation model improves significantly the prediction of transition onset location for flows at low Reynolds numbers with low level of free-stream turbulence. The key-point of new model is the pressure-gradient dependent threshold parameter $C_{NAT,crit}$. Despite the improvements achieved with this new model the correlations are still not perfect. Our numerical simulations indicate that the transition occurs too late at the upper side and too early at the lower side of the profile for higher angles of attack.

Acknowledgment

The work was partly supported by the institutional support RVO 61388998 and by the Czech Science Foundation under grant P101/12/1271.

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