ONE-DIMENSIONAL TEMPERATURE DISTRIBUTION OF CONDENSING ANNULAR FINS OF DIFFERENT PROFILES

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Abstract

A numerical approach based on the general differential equation results from an energy balance on an element of the fin has been developed and applied to the extended surfaces of different one-dimensional circular fin configurations (rectangular, convex parabolic, triangular and concave parabolic). The excessive fin surface temperatures relative to the temperature of surrounding air are plotted against the dimensionless fin distance measured from the fin tip. The effect of relative humidity on the fin surface temperature has been considered.

Keywords: heat exchanger, numerical method, wet fin analysis

1 Introduction

Extended surfaces or fins are frequently used in heat exchange devices to increase the surface area in order to enhance the rate of heat transfer. According to the heat exchanger application, the fin surface can be operated under fully dry, partially wet or fully wet conditions. The fin portion surface is considered wet if its temperature is below the dew point temperature of the incoming air. As a result, simultaneous heat and mass transfer occurs over this cooled portion of the fin. The condensate retained on the surface has hydrodynamic effects by changing the surface geometry and the air-flow pattern. Furthermore, a water layer on the surface increases local heat transfer resistance. In this case, the temperature over the fin surface changes simultaneously with the humidity ratio. Thus, the dry fin performance study is considerably different to the study of the same fin under condensing conditions. For any fin surface conditions, it is still difficult to find a universal equation relating to the evaluation of temperature profiles and the corresponding heat flows. Various types of fins have been used for a long time where attempts have been made to analyse the fin performance with and without condensation from the air-stream [1]-[12]. Under dehumidifying conditions, the fin-side energy balance cannot be formulated depending on temperature alone. A humidity ratio difference term also appears in the second order differential equation that describes the temperature distribution over the fin surface. However, it is very necessary to have a relationship between the humidity ratio of the saturated air on the fin surface and its corresponding temperature. Elmahdy and Biggs [5] presented a numerical method to get temperature distribution and overall fin efficiency of fully wet circular fin. They assumed a linear relationship between the humidity ratio of the saturated air and its corresponding temperature. Kazeminejad [12] obtained a numerical solution to the dimensionless temperature equation of a fully wet rectangular fin using the concept of sensible to total heat ratio. His differential equations were solved numerically using a shooting method which combines the Runge-Kutta method and the Newton-Raphson iteration. Rosario and Rahman [13] studied a radial fin assembly under partially wet, totally wet and totally dry conditions. Their computed results included the temperature distribution in the wall and the fin and the fin efficiency. Sharqawy and Zubair [14]

provided an analytical solution for temperature distribution and fin efficiency of a fully wet annular fin. The same authors [15] investigated different straight fin profiles (rectangular, triangular, hyperbolic and parabolic) under fully wet operating conditions. Similar to Elmahdy and Biggs [5], Sharqawy and Zubair [14]-[15] proposed another relationship between the humidity ratio of the saturated air on the fin surface and its corresponding temperature. They suggested that the maximum temperature at the fin tip for wet condition is the dew point temperature of the incoming air. Recently, Sharqawy et al [16] carried out a numerical analysis to study efficiency and temperature distribution of annular fins of constant and variable cross-sectional area under completely wet and partially wet operating conditions. Bourabaa et al [17] presented a numerical investigation of the fin efficiency and temperature distribution of a plain fin with combined heat and mass transfer. The second order differential equation that describes the temperature distribution along the fin surface has been solved using the finite difference scheme. Bourabaa et al [18] have used this method for the fully wet annular fin of constant fin thickness.

This study has been performed in order to simplify the general formulation of the conduction equation applied to the annular fin of various profiles. This numerical approach simplifies the general equation by avoiding the use of modified Bessel functions.

2 Mathematical formulation

Under wet fin surface the the total heat transferred to the surface is governed by the dual driving potentials of temperature and water vapor concentration. Fig. 1 shows straight fin with different shapes. The fin profile is defined according to the variation of the fin thickness along its extended length [19]-[20].

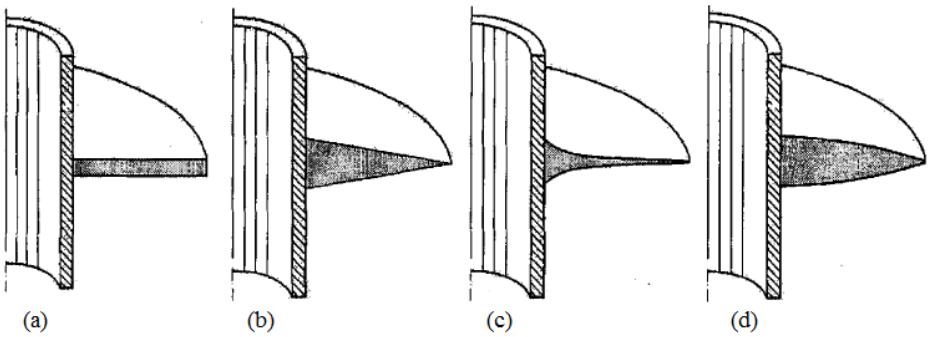

Figure 1: Schematic of different circular fin profiles: a. rectangular, b. triangular, c. concave parabolic, and d. convex parabolic

A finite difference method have been developed here to obtain solutions to the problem of the temperature distribution along the fin surface. The method has been developed for an annular fin of variable cross-sectional area. Referring to Fig. 2, a control volume of a fin of a length L is considered, the nodal temperatures are to be determined using N equally spaced nodes.

$$\Delta x = \frac{L}{N-1} \tag{1}$$

Under the following assumptions: 1) there is no indication of any change with time; 2) the temperature along the fin surface varies only in the x direction; 3) thermal

conductivity of fin material is constant; and 4) negligible radiation heat transfer, the finite difference formulation for a general interior node i is obtained by applying an energy balance on the volume element of this node. Let's write

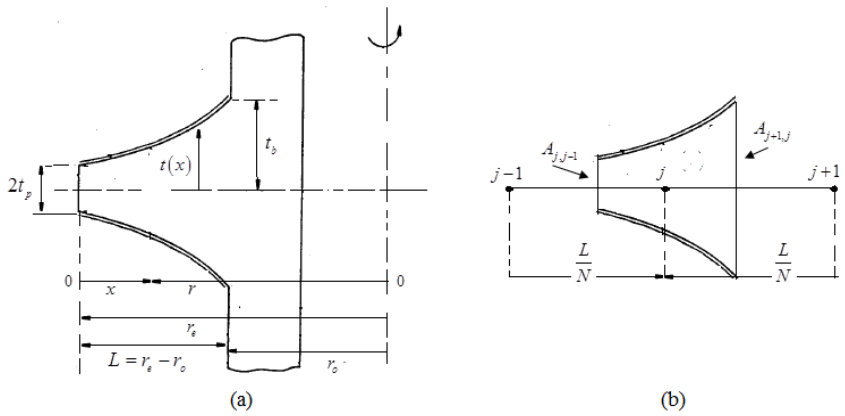

Figure 2: a. Fin thickness variation, and b. arbitrary internal nodes

$$kA_{j,j-1}\frac{T_{j-1}-T_j}{\Delta x}+kA_{j+1,j}\frac{T_{j+1}-T_j}{\Delta x}+h_cS_j\left[\left(T_a-T_j\right)+B\left(w_a-w_{sj}\right)\right]=0 \qquad (2)$$

Where, $A_{j,j-1}$ the conduction area at the boundary between local and downstream control volumes, $A_{j+1,j}$ the conduction area at the boundary between local and upstream control volumes, h_c the fin-side sensible heat transfer coefficient, k thermal conductivity of the fin material. S_j is the convection area at the jth control volume, T_a air stream temperature, T local temperature of the fin surface, w_a humidity ratio of the bulk air and w_{sj} the humidity ratio measured at T.

The parameter B is defined as:

$$B=\frac{i_{fg}}{c_pLe^{2/3}} \qquad (3)$$

In the above equation, the Lewis number Le is assumed one, c_p the isobaric specific heat and i_{fg} the latent heat of evaporation of water. The relationship that is suggested by Sharqawy and Zobair [14]-[15] between w_{sj} and T_j can be given by:

$$w_{sj}=a_2+b_2T_j \qquad (4)$$

Where

$$a_2=w_{s,b}+\frac{w_{s,dp}-w_{s,b}}{T_{dp}-T_b}T_b \qquad (5)$$

$$b_2=\frac{w_{s,dp}-w_{s,b}}{T_{dp}-T_b} \qquad (6)$$

Here, T_b and T_{dp} are the base fin and the dew point temperatures; respectively and, w_{sb} and $w_{s,dp}$ are the humidity ratios of saturated air evaluated at the base and dew point temperatures; respectively. Therefore, equation (2) can be rewritten as:

$$\alpha_iT_{j-1}+\beta_jT_j+\gamma_jT_{j+1}=\lambda_j \qquad (7)$$

Where

$$\alpha_i = 1;\ \beta_j = -\left[1 + \frac{A_{j+1,j}}{A_{j,j-1}} + \frac{h_c}{k}\frac{S_j}{A_{j,j-1}}\Delta x\left(1 + Bb_2\right)\right]$$

$$\gamma_j = \frac{A_{j+1,j}}{A_{j,j-1}};\ \lambda_j = -\left[\frac{h_c}{k}\frac{S_j}{A_{j,j-1}}\Delta x\left(T_a + B\left(w_a - a_2\right)\right)\right]$$

The variable conduction area is:

$$A(x) = 4\pi\left(r_e - x\right)t(x) \tag{8}$$

Where, r_e the outer fin radius, x the fin distance measured from the fin tip and $t(x)$ the fin half thickness at arbitrary location and given by:

$$t(x) = t_p + \left(t_b - t_p\right)\left(\frac{x}{L}\right)^n \tag{9}$$

Where, n is the fin profile exponent, t_b is the fin base half thickness and, t_p is the fin tip half thickness. For $t_p = 0$, equation (9) can be reduced to:

$$t(x) = t_b\left(\frac{x}{L}\right)^n \tag{10}$$

The base and the end conduction areas are, respectively:

$$A_b = 4\pi r_o t_b \tag{11}$$

$$A_e = 4\pi r_e t_b \tag{12}$$

In the case where $t_p = 0$, the conduction areas at the jth control volume are:

$$A_{j+1,j} = 4\pi\left(r_e - jdx\right)t_b\left(\frac{j}{N}\right)^n \tag{13}$$

$$A_{j,j-1} = 4\pi\left(r_e - (j-1)dx\right)t_b\left(\frac{j-1}{N}\right)^n \tag{14}$$

The convection surfaces are given by the following expression:

$$S_j = \frac{2\pi r_e^2 R}{N^2}\left(2N - \left(2j-1\right)R\right) \tag{15}$$

And

$$R = 1 - \frac{r_o}{r_e} \tag{16}$$

The finite difference equation for the first node, $i = 1$, is obtained by writing an energy balance on the volume element of length $L/(2N)$ at that boundary, again assuming heat transfer to be into the medium at all sides. It is noteworthy that, for a rectangular fin, no heat transfer occurs between the first node and the downstream control volumes.

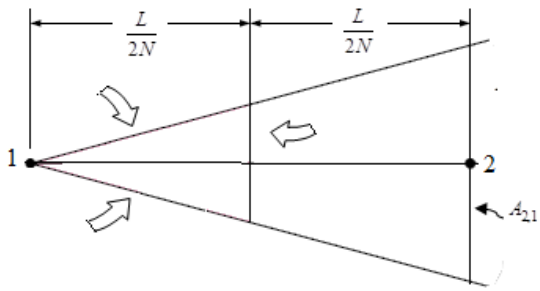

Figure 3: Schematic of the volume element of the first node

Referring to the Fig. 3, Eq. (7) can be reduced to:

$$\beta_1 T_1 + \gamma_1 T_2 = \lambda_1 \tag{17}$$

Where

$$\beta_1 = -\left[1 + \frac{h_c}{k}\frac{S_1}{A_{2,1}}\Delta x\left(1 + Bb_2\right)\right]; \gamma_1 = 1;$$

$$\lambda_1 = -\frac{h_c}{k}\frac{S_1}{A_{2,1}}\Delta x\left[T_a + B\left(w_a - a_2\right)\right]$$

For the end node, $i = N$, we have $T_{N+1} = T_b$. Substituting in Eq. (7), the following equation can be obtained

$$\alpha_N T_N + \beta_N T_N = \lambda_N + \gamma_N T_b \tag{18}$$

Where

$$\alpha_N = 1;\ \beta_N = -\left[1 + \frac{A_b}{A_{N,N-1}}\frac{h_c}{k}\frac{S_N}{A_{N,N-1}}\Delta x\left(1 + Bb_2\right)\right]$$

$$\gamma_N = -\frac{A_b}{A_{N,N-1}};\ \lambda_N = -\frac{h_c}{k}\frac{S_N}{A_{N,N-1}}\Delta x\left[T_a + B\left(w_a - a_2\right)\right]$$

Together, Eqs. (7), (17) and (18) constitute a system of N algebraic equations in N unknowns. The desired nodal temperatures can be easily found by solving them simultaneously.

This general formulation can be easily applied for dry fin surface conditions, just making some simplifications: $B \to 0$ and $h_c \to h_{dry}$.

The convective heat transfer coefficient on the fin-side under fully wet or fully dry conditions can be calculated using a specified correlation. Under condensing condition, correlations from Wang et al [21]-[22] and Pirompugd et al [23] can be used to evaluate the fin-side heat transfer coefficient. However for totally dry, the heat transfer coefficient can be calculated using correlations by Abu Madi et al [24] and Wang et al [25]-[26].

3 Results and discussion

The variations of fin temperature excess for different fin profiles are plotted against the dimensionless fin radius measured from the fin tip. Fig. 4 illustrates the variation of the fin surface temperature for different fin profiles under wet and dry fin surface conditions. Whether the surface is dry or wet, the temperature excess curves for convex

parabolic, triangular and concave parabolic fin shapes lie below that of rectangular profile. Under the same fin length, fin base thickness and operating conditions, the difference diminishes as X goes to 1. A further increase of fin length would result in an increase of conduction areas. These areas take the same conduction area of the rectangular fin as $X = 1$.

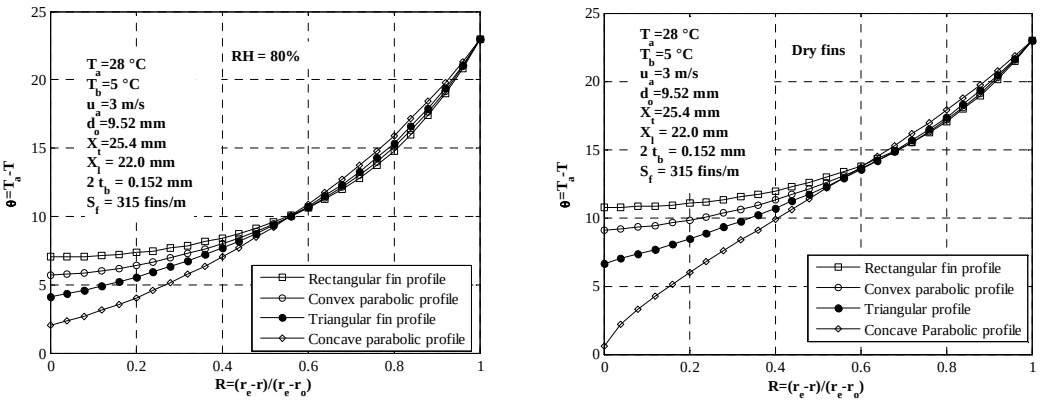

Figure 4: Temperature excess comparison: left- wet fins and, right- dry fins

For higher dimensionless fin radius, the temperature distribution curves for concave parabolic profile and triangular fin profile are slightly higher than other fin profliles. This can be attributed to the effect of the clearance C on the flow path shown in Fig. 5. At a small value of C, the front face of the fin receives a significant impingement flow which leads to higher sensible and latent heat transfer. This results in a higher fin surface temperature at lower value of the clearance.

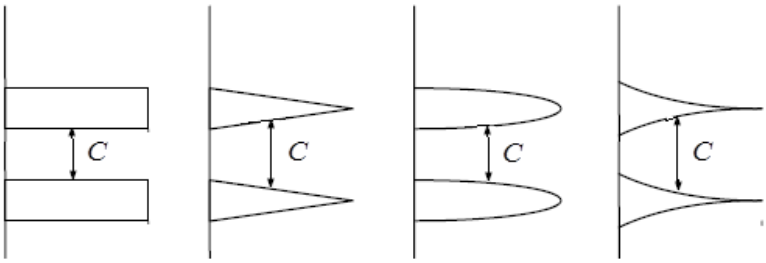

Figure 5: Effect of the clearance

Figures 6a-6d show the variation of the temperature excess against the dimensionless radius for three values of relative humidity (RH = 60, 80 and 100%) and compared with those under fully dry surface condition. The results here are in consistent with those from Elmahdy and Biggs [5] Kazeminejad [12], Sharqawy and Zubair [14]-[15] and Bourabaa et al [17]-[18]. They found that the temperature curves of a wet surface fin lie below those of dry surface fin. As the relative humidity increases, the departure of the temperature profile from the dry surface curve becomes greater. The increase of the air relative humidity is accompanied by an increase of the potential heat and mass transfer. In the other word, higher relative humidity results in a higher latent heat transfer and higher fin surface temperature.

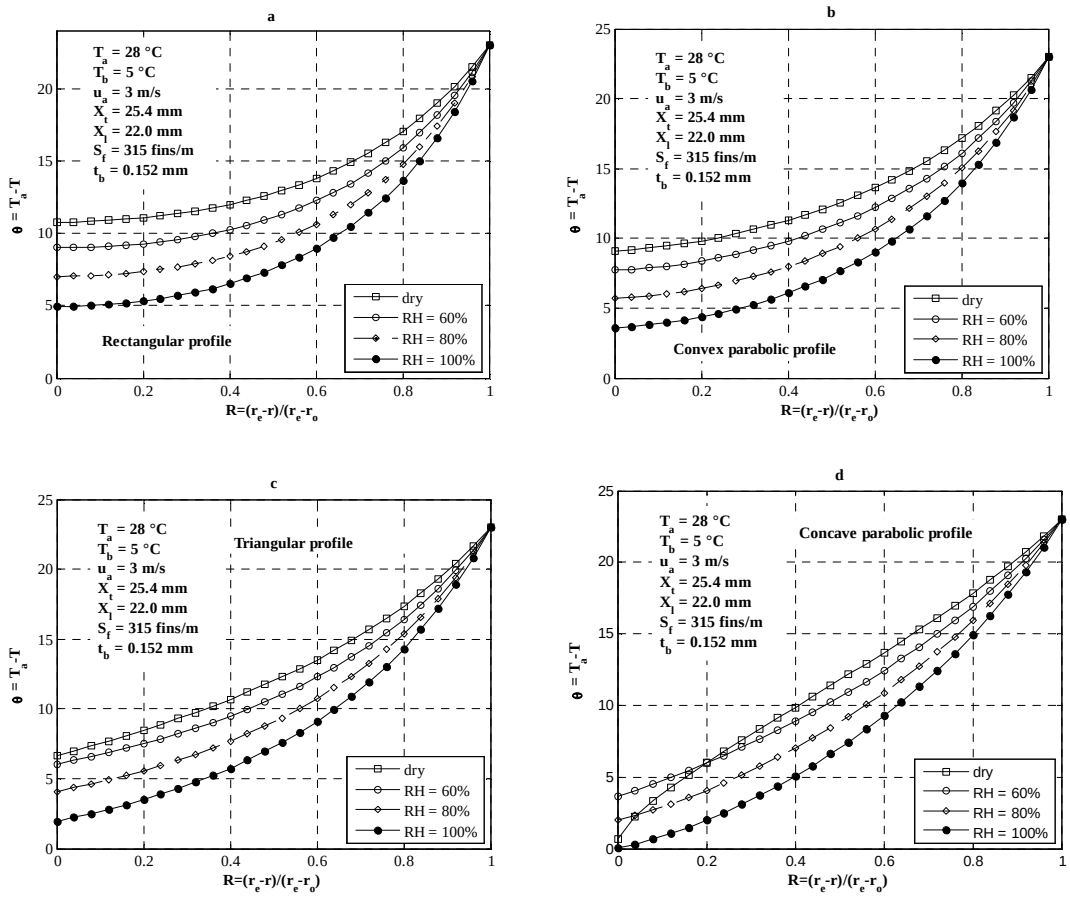

Figure 6. Effect of relative humidity on the temperature excess

For these values of relative humidity, Sharqawy and Zubair [14] found that the fin tip temperature for different fin shapes are below the dew point temperature of air. Conversely, the results here indicate that this is not always true. In order to explain the discrepancy, an investigation in the variation of the fin tip temperature against the dew point temperature (relative humidity between 50 and 100%) is carried out. The results are shown in Table. 1. As can be observed, the whole fin of different profiles is in partially dry condition for 50% relative humidity. The fin of rectangular profile becomes in fully wet as relative humidity increases to 60%. For the convex fin profile, the whole fin is in partially wet condition for this value of relative humidity and will be in fully wet conditions as the relative humidity is increased to 70%. The dry-wet boundary moves toward the fin tip with increasing the relative humidity value. However, the width of wet region does not increase by the same amount. The fins of triangular and concave fin profiles need highest relative humidity values to become completely wet, (80% for triangular profile and 100% for concave profile). Apparently, this is associated with the condensate blow off phenomena. The condensate is strongly attached to the surface of the rectangular fin. Conversely, for a concave fin a large amount of condensate can be easily leaving the surface dry. This discrepancy can also be attributed to the correlations used when calculating the heat transfer coefficients. Note that the heat transfer coefficients for wet and dry conditions used in the present work are that derived from Wang et al [21] and [25].

Table 1. Fin tip temperature.

RH	T_{dp}	Rectangular	Convex	Triangular	Concave
		T_{tip}	T_{tip}	T_{tip}	T_{tip}
50	16.63	18.26	19.58	21.35	24.20
60	19.52	19.00	20.29	21.94	24.35
70	22.04	19.97	21.25	22.88	25.07
80	24.25	20.98	22.29	23.92	25.99
90	26.23	22.01	23.35	24.99	26.97
100	28.00	23.03	24.40	26.05	27.96

4 Conclusion

The one-dimensional conduction equation applied to the annular fins of arbitrary profile (rectangular, convex parabolic, triangular and concave parabolic) has been numerically solved. It is found that the fin surface temperature increases with increasing relative humidity. This implies that the contribution of mass transfer has an importance effect on the surface temperature distribution when dehumidification of moist air occurs. In addition, the fin tip temperature is not always below the dew point temperature of moist air. The fins of triangular and concave profiles need highest relative humidity to become completely wet.

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