NUMERICAL STUDY OF STRATIFICATION EFFECTS ON LOCAL WIND FLOW AND POLLUTION DISPERSION

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Abstract

The aim of this paper is to demonstrate and clarify the various effects of stable stratification on the wind flow over wall mounted obstacles in atmospheric boundary layer. The example being solved is motivated by the wind flow and pollution dispersion studies in the proximity of various technological blocks in the area of opencast coal mine. The equations of fluid motion are based on the Boussinesq approximation of momentum equations, supplemented by the divergence-free incompressibility constraint. Turbulence is described by a modified $SSTk-\omega$ model. Concentration of heavy particles is considered and described by an additional transport equation.

Keywords: atmospheric boundary layer, stratification, turbulence, $SSTk-\omega$ model, finite-volume

1 Introduction

The dispersion of heavy particles is a limiting factor in many areas of human industrial and agricultural activity. Thus it is very important to be able to predict the local concentration of airborne aerosol in order to quantify and possibly reduce the associated health risks. This study is motivated by the pollution dispersion study over an open shelter covering a technological device (block) producing high levels of heavy particles. These particles are carried by the wind passing through the open shelter. The aim was to propose some simple means of reduction of the particles emission and their consequent transport to surrounding areas.

Although it is possible to investigate the local flow field experimentally, using a scaled down model in a wind tunnel, the effects of thermal stratification are rather hard to or even impossible to be included in such laboratory measurements. This is why the mathematical modeling and numerical simulations of this problem are an important supplement to detailed experimental investigation.

To evaluate the effects of stratification on the local flow field and pollution dispersion a simplified 2D geometry, possessing the main features of realistic case was adopted. The 2D problem is solved in a domain Ω, which is a subset of $x-z$ plane. The problem setup is shown in the figure 1. The domain dimensions are $100m \times 50m$. The shelter size is set to width $W = 10m$ and height

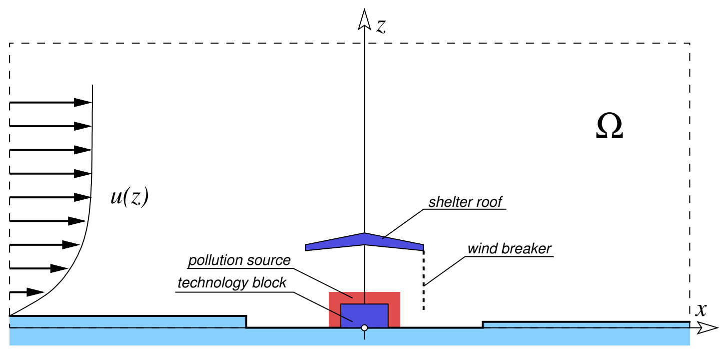

Figure 1: Geometrical configuration of the problem.

$H = 8m$. The case with impermeable wind breaker barrier is presented in this paper, however a range of semi-permeable (net-like) wind breakers was simulated.

2 Mathematical model

Flow and turbulence model

The Boussinesq approximation for an incompressible, thermally stratified fluid flow is considered. The unknown fields are the velocity $\boldsymbol{u}$, pressure p, potential temperature Θ (resp. its perturbation p', Θ')

$$\nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\frac{\nabla p'}{\rho_*} + \frac{1}{\rho_*}\nabla \cdot \Big[\mu(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)\Big] - \frac{\Theta'}{\Theta_*}\boldsymbol{g} \tag{2}$$

$$\frac{\partial \Theta'}{\partial t} + \boldsymbol{u} \cdot \nabla\, \Theta' = \nabla \cdot \big[\widetilde{\kappa}\, \nabla \Theta'\big] - \boldsymbol{u} \cdot \widetilde{\boldsymbol{\gamma}} \tag{3}$$

Here $\widetilde{\kappa}$ is the thermal diffusivity and $\widetilde{\boldsymbol{\gamma}} = \mathrm{grad}\Theta_0$ is related to the thermodynamic lapse-rate of atmosphere. For turbulent flow, the (laminar) viscosity and thermal diffusivity are increased by their turbulent counterparts, i.e. $\mu \longrightarrow K_u = \mu + \mu_T$, resp. $\widetilde{\kappa} \longrightarrow K_\Theta = \widetilde{\kappa} + \widetilde{\kappa}_T$. The same approach is also applied to the pollutant concentration diffusivity $\varkappa$ in the equation (15).

To take into account the thermally generated turbulence the standard $SSTk - \omega$ model was modified to include the buoyancy effects.

$$\rho\left(\frac{\partial k}{\partial t} + \boldsymbol{u} \cdot \nabla k\right) = \big(P + G_{bk}\big) - \beta^* \rho k \omega + \nabla \cdot \left[\Big(\mu + \frac{\mu_T}{\sigma_k}\Big)\nabla k\right] \tag{4}$$

$$\rho\left(\frac{\partial \omega}{\partial t} + \boldsymbol{u} \cdot \nabla \omega\right) = \frac{\chi\rho}{\mu_T}\big(P + G_{b\omega}\big) - F_4 \beta \rho \omega^2 + \nabla \cdot \left[\Big(\mu + \frac{\mu_T}{\sigma_\omega}\Big)\nabla \omega\right] + 2\rho\frac{1 - F_1}{\sigma_{\omega 2}\omega}\nabla k \cdot \nabla \omega \tag{5}$$

The turbulent kinetic energy production is computed using the strain-rate tensor $\boldsymbol{S}$ according to:

$$P = \sum_{i,j=1}^{3}\Big(2\mu_T S_{ij} - \frac{2}{3}\delta_{ij}\rho k\Big)\frac{\partial u_i}{\partial x_j} \qquad \text{with} \qquad S_{ij} = \frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\Big) \tag{6}$$

In the original Wilcox's $k - \omega$ model the turbulent viscosity was computed from a simple formula $\mu_T = \rho k/\omega$. The Stream Stress Transport (SST) model uses the Bradshaw assumption that in the boundary layer the principal stress is linearly proportional to the turbulent kinetic energy as $\tau = a_1 \rho k$. The SST limitation is explained in detail in the report [6]. The turbulent viscosity is thus computed from:

$$\mu_T = \frac{a_1 \rho k}{\max(a_1\omega; |\boldsymbol{S}|F_2)} \qquad \text{where} \qquad |\boldsymbol{S}| = \sqrt{2\boldsymbol{S} : \boldsymbol{S}} \qquad \text{and} \qquad a_1 = 0.31 \tag{7}$$

The production of turbulent kinetic energy is affected by buoyancy (stratification). This effect can be included using an extra production term in the equation for k.

$$G_{bk} = -\frac{\mu_t}{\rho \sigma_\rho}\boldsymbol{g} \cdot \nabla \rho \tag{8}$$

In the case of stable stratification, when the density gradient $\nabla \rho_0$ points in the same direction as the gravity acceleration vector $\boldsymbol{g}$, the term G_{bk} is negative, i.e. the TKE is inhibited by the background stratification. For the Boussinesq formulation using potential temperature Θ rather than density, the extra TKE production term can be rewritten as:

$$G_{bk} = \frac{\mu_t}{\Theta_* \sigma_\Theta}\boldsymbol{g} \cdot \nabla \Theta \tag{9}$$

In the case of Boussinesq approximation, this term can be further expanded to separate the background and local temperature gradients

$$G_{bk} = \frac{\mu_t}{\Theta_* \sigma_\Theta} \boldsymbol{g} \cdot (\boldsymbol{\nabla}\Theta_0 + \boldsymbol{\nabla}\Theta') = \frac{\mu_t}{\Theta_* \sigma_\Theta} \boldsymbol{g} \cdot (\gamma + \boldsymbol{\nabla}\Theta') = \frac{\mu_t}{\Theta_* \sigma_\Theta} (\boldsymbol{g} \cdot \boldsymbol{\nabla}\Theta' - g\gamma) \tag{10}$$

Here we have used the fact that the vectors $\boldsymbol{g}$ and $\boldsymbol{\gamma}$ are parallel and gravity acceleration is negative (in the chosen coordinate system).

The extra source term is always used in the balance of equation for k. The role of this term in the equation for dissipation, i.e. ϵ, resp. ω is not so well understood. In some cases the term is included, i.e. $(P + G_{b\epsilon})$ is used instead of P also in the ϵ balance, while sometimes it is neglected there. In some cases its contribution is considered in a flow-dependent way that accounts for the angle between the local velocity vector and the gravity acceleration vector. It means that only part of the G_{bk} is used in $G_{b\epsilon}$.

$$G_{b\epsilon} = G_{bk} \tanh \frac{\|\boldsymbol{u}_{\parallel}\|}{\|\boldsymbol{u}_{\perp}\|} \tag{11}$$

where $\boldsymbol{u}_{\parallel}$ and $\boldsymbol{u}_{\perp}$ refers to the components of the velocity vector that are either parallel or orthogonal to the gravity vector. It means that in the case the velocity is aligned with the gravity vector, i.e. $\boldsymbol{u} \parallel \boldsymbol{g}$, $G_{b\epsilon} \to G_{bk}$, while for $\boldsymbol{u} \perp \boldsymbol{g}$, $G_{b\epsilon} \to 0$. It can be shown [4] that the buoyancy contribution to the balance of ω is:

$$G_{b\omega} = \frac{1+\chi}{\chi} G_{b\epsilon} - \frac{1}{\chi} G_{bk} \tag{12}$$

Pollution dispersion model

The pollution dispersion model is based on macroscopic Eulerian description of concentration of heavy particles. The balance equation includes advection of particles, their turbulent diffusion. The governing equation can be written in the non-conservative differential form as

$$\frac{\partial C}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla} C = \boldsymbol{\nabla} \cdot [\varkappa \, \boldsymbol{\nabla} C] \tag{13}$$

The gravitational settling of heavy particles can be included using the assumption that the vertical velocity of particles differs from the wind velocity by a value that can be approximated by a Stokes formula:

$$w_s = \frac{(\rho_p - \rho_f)}{18\mu} g D_p^2 \tag{14}$$

Here ρ_p is the particle density, ρ_f is the fluid density, μ is the (laminar) viscosity, g stands for gravity acceleration and D_p is particle diameter. The resulting settling velocity w_s alters the corresponding concentration advection velocity w in the transport equation:

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} + (w - w_s) \frac{\partial C}{\partial z} = \mathrm{div}[\varkappa \, \mathrm{grad} C] \tag{15}$$

Numerical solver

The whole model is solved using a finite-volume method on a structured grid. An explicit multistage Runge-Kutta method was used for time integration. See e.g. [5] for details. Non-linear digital filters are used to avoid non-physical oscillations and stabilize the computational process [1]. The solid obstacles (roof, rectangular block, impermeable barrier) are simulated by a simple immersed boundary approach, where the velocity is set to zero inside of these obstacles. This allows to solve complex configurations on simple orthogonal grids.

3 Numerical results

Numerical simulations were performed for the case without stratification (i.e. neutral stratification with $\gamma = 0.0\, K/m$), compared to stably stratified case where $\gamma = 1.0\, K/m$ was prescribed in near ground layer of the depth $10m$. The wind velocity profile was prescribed at the inlet as a power law profile with the exponent $\alpha = 0.13$ and maximum velocity of $u = 5m/s$ attained at the upper boundary (at height $50m$).

The (stable) thermal stratification, represented by the background temperature gradient γ, generates local perturbations in potential temperature due to the source term in the equation (3) of the form $-w\gamma$. This means that positive vertical velocity brings cold air to upper layers and thus generates negative temperature perturbation. Opposite effect is connected with descending flows (e.g. in the proximity of the wind breaker in this case), leading to positive local perturbation in the temperature profile. This effect is clearly demonstrated in the Figure 2, where the flow pattern is shown together with potential temperature perturbation contours..

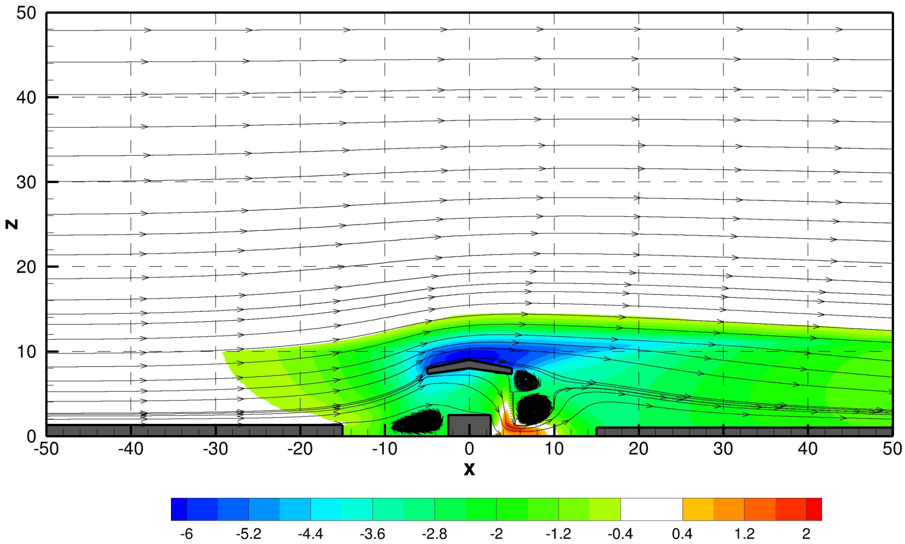

Figure 2: Potential temperature perturbation.

On the other hand, the potential temperature perturbation affects the flow via the buoyancy term in the momentum balance (2). The negative temperature perturbation acts against the (positive) vertical velocity and thus tends to suppress vertical motion. This effect, although rather weak at this scale, is visible in the Figure 3.

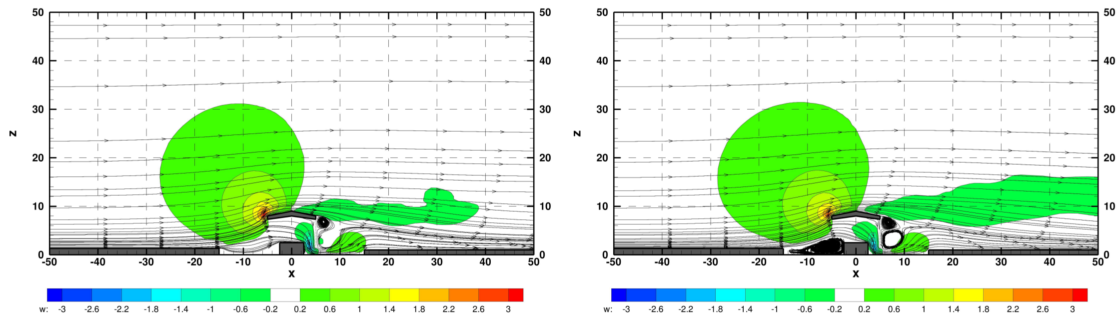

Figure 3: Vertical velocity contours for neutral (left) and stably stratified (right) flow.

The most notable effect of stable stratification in this case is the change in the turbulent kinetic energy k. Typically the stable stratification leads to inhibition of turbulence and thus the values of k are lower in general. The only exception are the situations where the descending flow brings the warmer air into lower layer, which causes locally unstable temperature gradient (i.e. the temperature decreases with height). In these regions the turbulence is generated due to temperature gradient as it follows from the extra source term G_{bk} (see equations (9,10)) in the TKE balance.

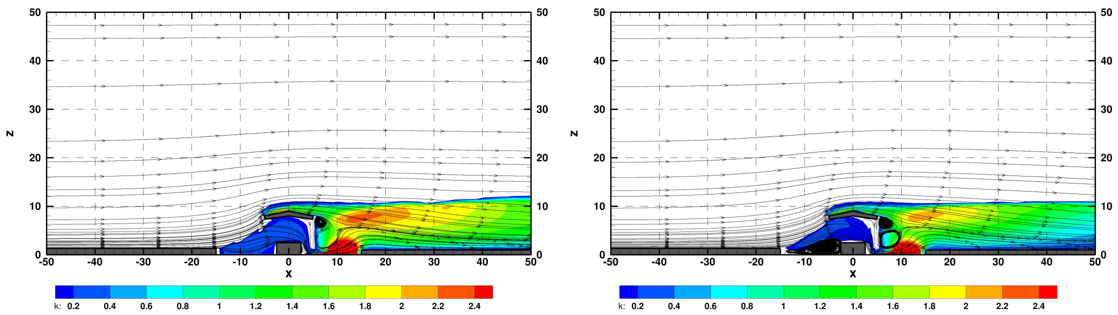

Figure 4: Turbulent kinetic energy k for neutral (left) and stably stratified (right) flow.

This effects can be seen in the Figure 4 and explained from the local flow patterns and temperature distribution shown in Figure 2.

The above described stratification induced variations in the flow and turbulence fields manifest itself in the pollution dispersion. The dispersion of heavy (brown coal dust) particles of the size $50\mu m$ is compared in the Figure 5. The volume source is assumed in the close proximity of rectangular block under the shelter. The pollutant is carried by the wind flow into surrounding area outside the shelter.

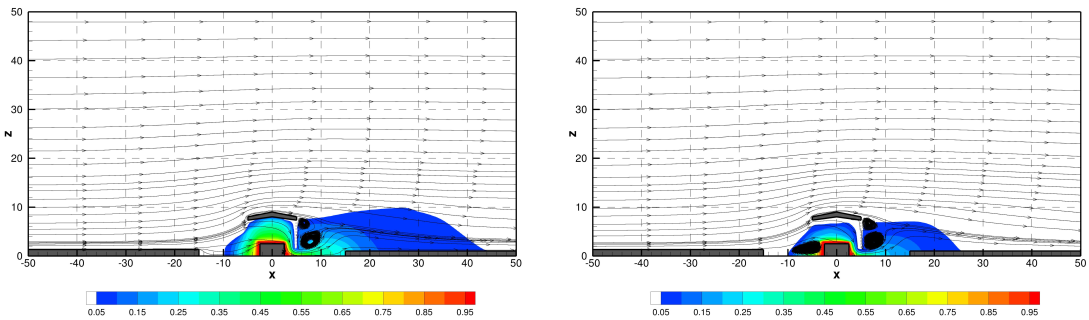

Figure 5: Pollutant concentration contours for neutral (left) and stably stratified (right) flow.

It is obvious that although the flow patterns are very similar in both cases, the differences in TKE are responsible for non-negligible alteration (reduction in this case) of the pollutant concentrations. It means the concentrations are reduced and smaller area is affected by pollution under the stable stratification.

4 Conclusions & Remarks

The present numerical simulations show that at the given scale the flow field changes (at realistic temperature gradients) are rather small. More important seems to be the inhibition of turbulence due to stable stratification. This can potentially lead to reduction of pollution dispersion. There still remain an open question how the unstable stratification or local thermal convection (e.g. due to solar heating of the shelter roof) can affect the pollution dispersion in similar cases.

Future research in this area will focus on validation of the newly developed computer code using the experimental data (for thermally neutral case) obtained for identical geometry in the wind tunnel of the Institute of Thermomechanics of the AS CR.

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