AN EXPERIMENTAL STUDY OF THE LAMINAR-TURBULENT TRANSITION IN A TILTED TAYLOR-COUETTE SYSTEM SUBJECT TO FREE SURFACE EFFECT

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Abstract

An experimental study of the laminar – turbulent transition between two coaxial rotating cylinders with the inner cylinder rotates and outer one stationary is presented in this paper. Special attention is given to the onset of various flow modes in tilted and partially filled system. The flow behaviour, the laminar turbulent transition and the features of different regimes are discussed for different inclination angles, filling ratio and Taylor numbers.

It is established that the different filling ratio and the inclination angle of the system deeply affect the flow patterns. Furthermore, the most significant result concerns the relaminarization of the flow when the aspect ratio is decreased and inclination angle is increased for a given value of Taylor number.

Keywords: laminar turbulent transition, free surface, tilted system, relaminarization phenomenon, Taylor-Couette flow

1 Introduction

The fluid flowing in annulus between two concentric rotating cylinders with one cylinder can be set to rotate or both cylinders rotate, termed Taylor – Couette flow (TCF), has been intensively investigated, both fundamental and applied. Since the seminal works of Couette [1] and Taylor [2] this flow has received potential interests because of its importance in the hydrodynamic stability theory as well as the laminar turbulent transition.

Further, The Taylor Couette flow has been the subject of numerous investigations as it is crucial to many technical applications; tribology, filtration, rotating machinery, catalytic reactions.

Due to the various flow regimes existing between laminar turbulent transitions, a large body of literature focused on this flow for over a century by physicist and mathematicians.

Couette [1] has the first who measured the dynamic viscosity of liquid contained between two rotating cylinders where outer cylinder rotates and inner one held stationary.

However, the stability of the flow between two concentric cylinders has been determined by Taylor [2], both analytically and experimentally, for a narrow gap. Their work showed excellent agreement between theoretical analysis and experimental measurement. Later, Chandrasekhar [3, 4] presented an extensive study of the flow stability problem in Taylor Couette system for a small and wide gap.

The first experimental investigation demonstrating the existence of more than one pattern in Taylor Couette flow has been made by Coles [5]. Moreover, he was the first that observed and reported the regime of wavy vortex flow.

Gollub and Swinney [6] and Fenstermacher et al [7] have presented several experimental investigations of the laminar turbulent transition in the Taylor-Couette system. They showed that successions of instabilities are enough to drive the flow from laminar state to chaotic regime with increasing the angular velocity of the inner cylinder.

In addition, Burkhalter and Koschmieder [8] measured the wavelength of axisymmetric vortices from their initiation up to values as large as $80Ta_{c1}$. They have reported that the wavelength of the vortices in the annular gap can be changed by changing the initial conditions.

Bouabdallah [9] presented interesting analytical and experimental studies to determine the various flow regimes existing between laminar and turbulent flow by using the chaos theory and polarographyic technique.

So far, a substantial experimental measurements and theoretical works have been carried out regarding the transition zones between laminar Couette flow and turbulent flow. In addition, the flow behaviour and various bifurcations have been analyzed and detailed by numerous authors for different boundary conditions, among these are Donnelly [10], Marcus [11], Jones, [12], Andereck *et al* [13] ,Takeda *et al* [14], Wereley and Lueptow [15], Mullin *et al* [16], Abcha *et al* [17], Borrero and Schatz [18], Martinez-Arias *et al* [19].

On the other hand, the presence of the free surface in the Taylor Couette system profoundly affects the flow patterns and the occurrence of the various instabilities.

Mahamdia et al [20] used the polarographic technique to present the experimental results of the effects of the free surface and aspect ratio on the behavior of flow between two concentric cylinders. They showed the existence of a critical height of the column of liquid for which the laminar turbulent transition occurs without the wavy mode.

Recently, Watanabe and Toya [21] and Watanabe et al [22] have investigated, both numerically and experimentally, the effect of the free surface on the occurrence of the Taylor vortices in the annulus between to coaxial rotating cylinders. They showed that anomalous modes with outer flow near the bottom end wall or inner flow at the free surface appear in some conditions.

From the foregoing discussion, it is clear that substantial experimental and theoretical works have been published for the regime of transitions. However, only few studies have been reported on the flow modes with free surface. In the present work, an experimental attempt has been made to investigate the combined effect of inclination of the system and the free surface on the flow patterns. In this study, we have tested a wide range of the inclination angle and filling ratio of the system.

2 Experimental setup, visualization techniques and procedures

2. 1. Taylor – Couette apparatus

The experimental device consisting of two coaxial rotating cylinders are made of the plexiglass. The outer cylinder is kept stationary while the inner one rotates with angular velocity Ω_1. The system is planned for the use of several interchangeable cylinders and designed for to facilitate the operations of assembling and disassembling, as shown in figure 1. The geometry is characterised by the following parameters:

- The radius ratio $\eta=R_1/R_2=0.909$ ($R_1=50\pm0.2$mm and $R_2=55\pm0.2$mm).
- The aspect ratio $\Gamma=H/d$ which is variable

The control parameter that defines the various flow regimes is Taylor number:

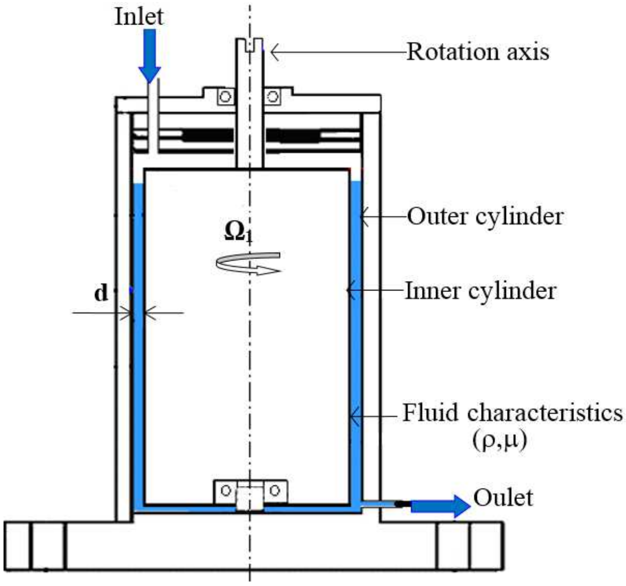
$$Ta=\frac{\rho\Omega_1 R_1 d}{\mu}\sqrt{\frac{d}{R_1}}$$

The Taylor number was increased stepwise by a quasi-static increase of the rotating velocity of inner cylinder following the relationship:

$$\frac{\Delta\Omega_1}{\Omega_1}\leq 1\%$$

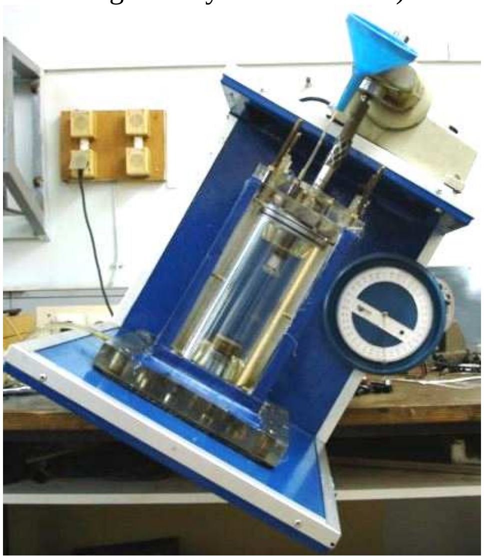
Where $\frac{\Delta\Omega_1}{\Omega_1}$ is rate of increase.

For Ω_1 characterizing the appearance of a given phenomenon, we notice measurements characteristic of the structures considered and taking a picture of the state of the flow stored and treated by means of appropriate logiciels with PC.
The working liquid is composed of 20% of Vaseline oil with 80% of ether petroleum. The flow was made visible by aluminum flakes as a light-diffusing impurity added in a small amount (1 g/l). The density and dynamic viscosity are, respectively, ρ=785 kg/m^3 and μ=4.9×10^{-3} Pa.s measured at T=22° C. The temperature is measured by an electronic captor (−50 to +150°C) with accuracy less than 1/10°C.



a) Taylor Couette geometry

b) Vertical system



c) Tilted system

Figure 1: Taylor – Couette apparatus

2. 2. Visualization techniques:

The visualization of flow was implemented by two methods as follow:

- Visualization by the reflexion of light, which is proceeded by reflexion of a beam of light diffused by front an outside source on the flow showing the mode and structure associated to the movement, as illustrated in the Figure 2.

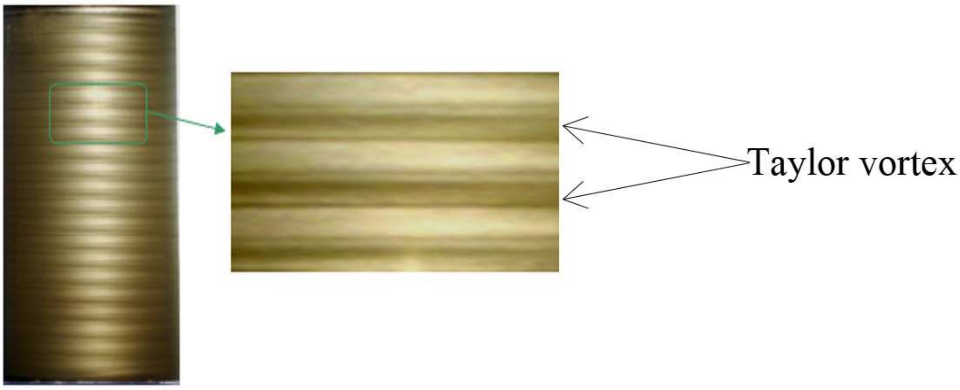


Figure 2: Visualization by optical reflexion

- Visualization by transverse optic transmission, which is based on the optical transmission of a beam luminous coming from a source placed at the opposite of the observer and crossing the whole of the flow. This mode of lighting makes possible to visualize the in-depth structure of the movement related to the shape of the cell, as illustrated in Figure 3.

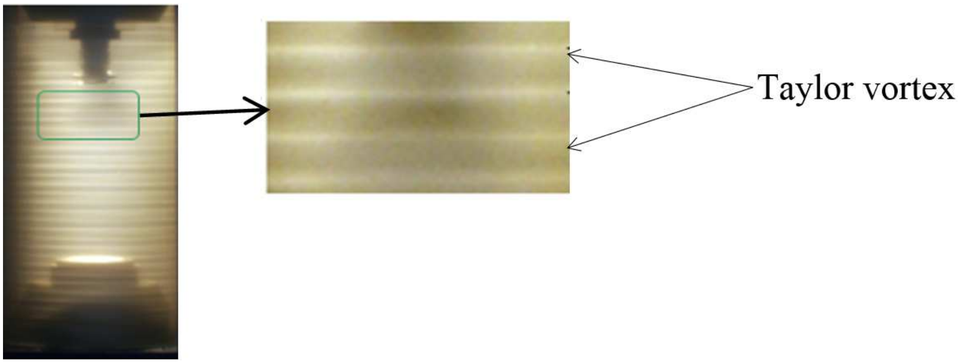


Figure 3: Visualization by optical transmission

2. 3. Procedure

2. 3. 1. First Procedure

For the flow system completely filled, we fixed the Taylor numbers corresponding to the onset of different flow modes, i.e., TVF, WVF, MWVF and chaos and then we varied the inclination angles to studying the flow behavior. On the other hand, when the system is partially filled, we have examined only the first and second instabilities (TVF and WVF).

2. 3. 2. Second procedure

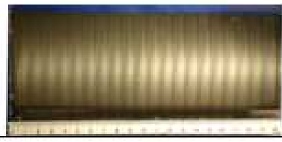
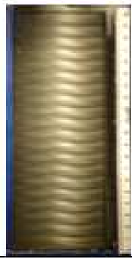
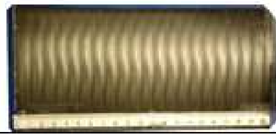
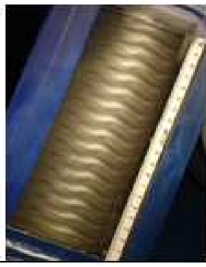
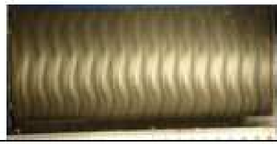
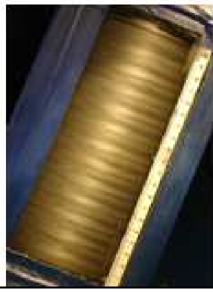
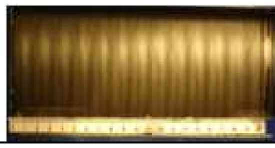
In this procedure, we fixed the inclination angle and filling ratio and we varied the angular velocity of inner cylinder in order to determine critical values of Taylor number ,i.e., Ta_{c1}, Ta_{c2}, Ta_{c3} and Ta_{c4} corresponding to the occurrence of various flow regimes.

3. Results and discussion

3.1. System completely filled

The observations carried out show that if the system is completely filled $\Gamma = \Gamma_{max} = 40$, the inclination angles have no effect on the flow behaviour for the various instabilities (TVF, WVF, MWVF and CF), as shown in table 1.

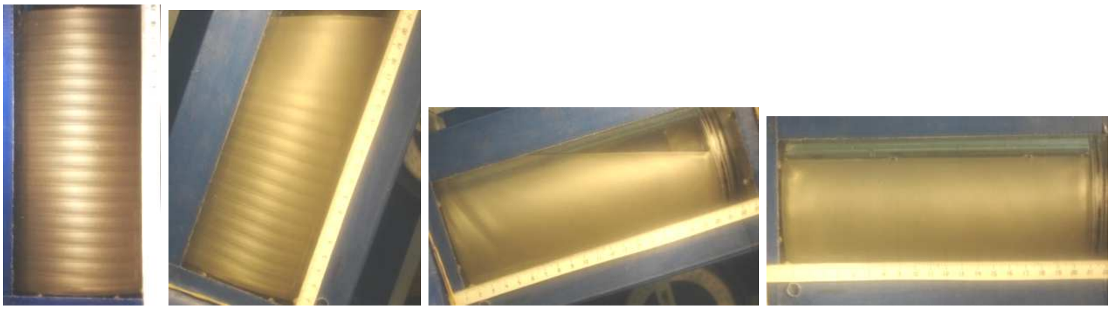
Table 1: Effect of inclination on the onset of various regimes for completely filled system $\Gamma = \Gamma_{max} = 40$

Inclination angles / Flow modes	0	30°	90°
TVF $Ta_{c1} = 42$			
WVF $Ta_{c2} = 49$			
MWVF $Ta_{c3} = 390$			
Chaos $Ta_{c4} = 710$			

3. 2. System partially filled

3. 2. 1 Taylor vortex flow

When the system is partially filled $\Gamma < \Gamma_{max}$, we have examined the inclination effect on the flow behaviour for two regimes; TVF and WVF respectively. We increase slowly the angular velocity of inner cylinder until the appearance of the TVF at $Ta_{c1} = 44 \pm 3$, then we proceed to the inclination of the system from 0 up to 90°. As can be seen in Figure 4, the disappearance of the Taylor vortices is made gradually near the free surface with increasing of the inclination angles. The flow relaminarization phenomenon is appeared for a given critical value of $\alpha = \alpha_c$.

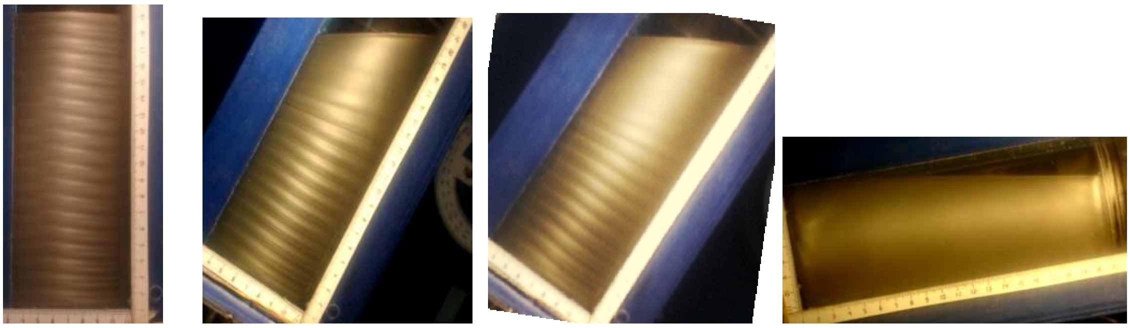


α= 0° α= 30° **α_c= 70°** α= 90°

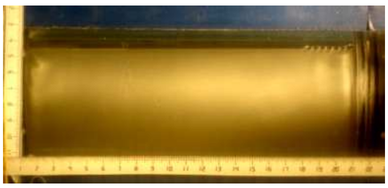
Figure 4: Effect of inclination α on Taylor vortex flow at ***Ta* = *44±2*** for partially filled system (***Γ*** **= 37**), relaminarization of the flow at $\boldsymbol{\alpha_c}$ **= 70°**

3. 2. 2 Wavy vortex flow

The figure 5 depicts the influence of combined effects of the filling ratio and inclination angle on the wavy vortex flow. We observe that the WVF is progressively attenuated according to inclination angle. For α=αc', the travelling waves disappear completely and the WVF returns to the TVF. With more increased the inclination angle up to $\alpha = \alpha_c$ the flow becomes completely laminar (relaminarization phenomenon).



α= 0° α= 30° **α'_c= 35°** **α_c= 80°**



α= 90°

Figure 5: Effect of inclination on the wavy vortex flow at ***Ta* = 56±3** for partially filled system (***Γ*** **= 37***)*.Disappearance of Wavy mode at $\boldsymbol{\alpha_c'}$ **= 35°** and relaminarization of the flow at $\boldsymbol{\alpha_c}$ **= 80°**

For other filling ratio, the critical angles of relaminarization α_c and critical angles of disappearance of wavy mode α'_c are shown in the following figure 6 and 7.

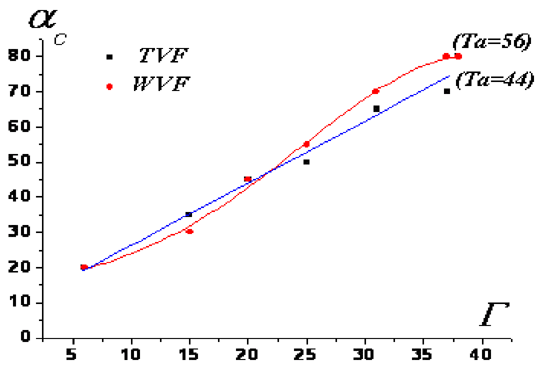


Figure 6: Critical inclination angles of the relaminarization phenomenon α_c versus filling ratio Γ

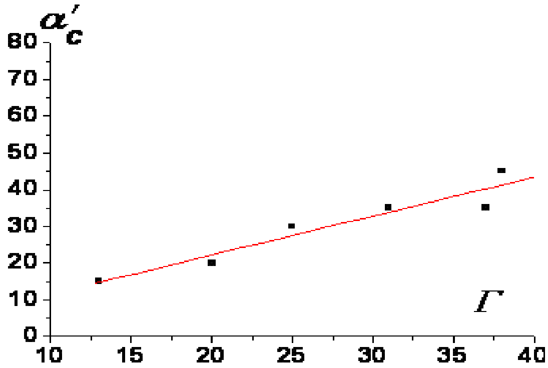


Figure 7: Critical angle inclination angles α'_c of the disappearance of WVF versus filling ratio Γ at Ta_{c2} = 3

We note that for a system partially filled, this experimental study showed up the coexistence of three flow modes in the Taylor–Couette system, as follow:

- **Zone 1:** Laminar Couette Flow (LCF)
- **Zone 2:** Mode of the Taylor Vortices with warping (compression and expansion of the waves).
- **Zone 3:** Mode of the Taylor vortices without warping as a vertical Taylor–Couette flow

In addition, we observed the appearance of a laminar zone at the free surface. This is due to the disappearance of Taylor vortices near the higher part of the system. However, we noted that the zone adjacent the laminar zone consists of Taylor vortex inclined. This flow is characterized by a size of the vortices which is variable in the same circumference. This situation is physically remarkable with a compression of waves on a side and an expansion of waves at the opposite side for the same value of Ta, as shown in the figure 8.

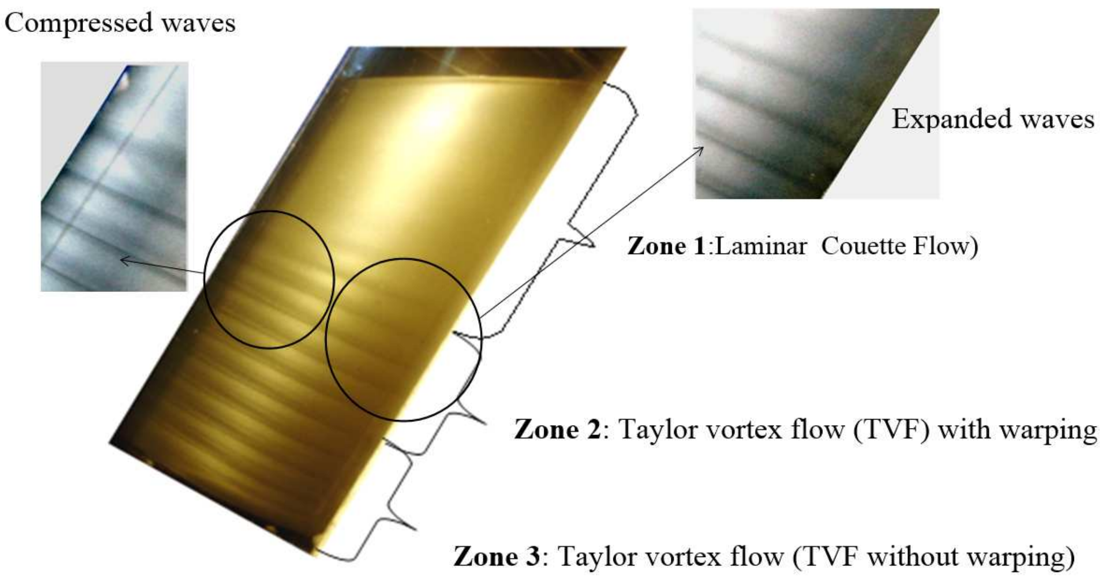


Figure 8: Coexistence of three flow modes in the Taylor–Couette Flow (Ta = 44±3, α = 30±0.5°)

In the foregoing discussion, we have examined the effects of α and Γ on the flow patterns for the case where the Taylor number is fixed in advance (Ta=44 and Ta=56). For example, to given the value of Ta_{c1} at fixed Γ, we vary α until to reach $\alpha=\alpha_c$ that is to say an angular critical value which characterizes the relaminarization of the flow.

In otherwise, we fixed the inclination angle and filling ratio with varying the angular velocity in order to determine critical values of Taylor number for different flow regimes (Ta_{c1}, Ta_{c2}, Ta_{c3} and Ta_{c4})

The examination of the experimental results allowed us to analyze the effect of the filling ratio and the inclination angle which leading to determine the phenomenological laws as shown in the relationships 1 and 2 :

- Linear low: Relation valid for Γ_{max} = 40 (completely filled)

$$Ta_c(\alpha^*) = A \qquad (1)$$

For the flow system completely filled the experimental test allows us to find the following critical values: Ta_{c1} = 42±2, Ta_{c2} = 48±4, Ta_{c3} = 390±8 and Ta_{c4} = 740. In addition, we have found that the various flow regimes are independent of the inclination angle. We note also that the curves showing the evolution of critical Taylor numbers versus the inclination angle are straight without slope. This result is probably due to the confinement of the flow by the end walls.

- Exponential low

$$Ta_c(\alpha^*) = A + B\exp(\alpha^*/C) \qquad (2)$$

Relation valid for $\Gamma < \Gamma_{max}$ (partially filled). The constants A, B, C are given in table 1, which are determined from the curves in figure 9.

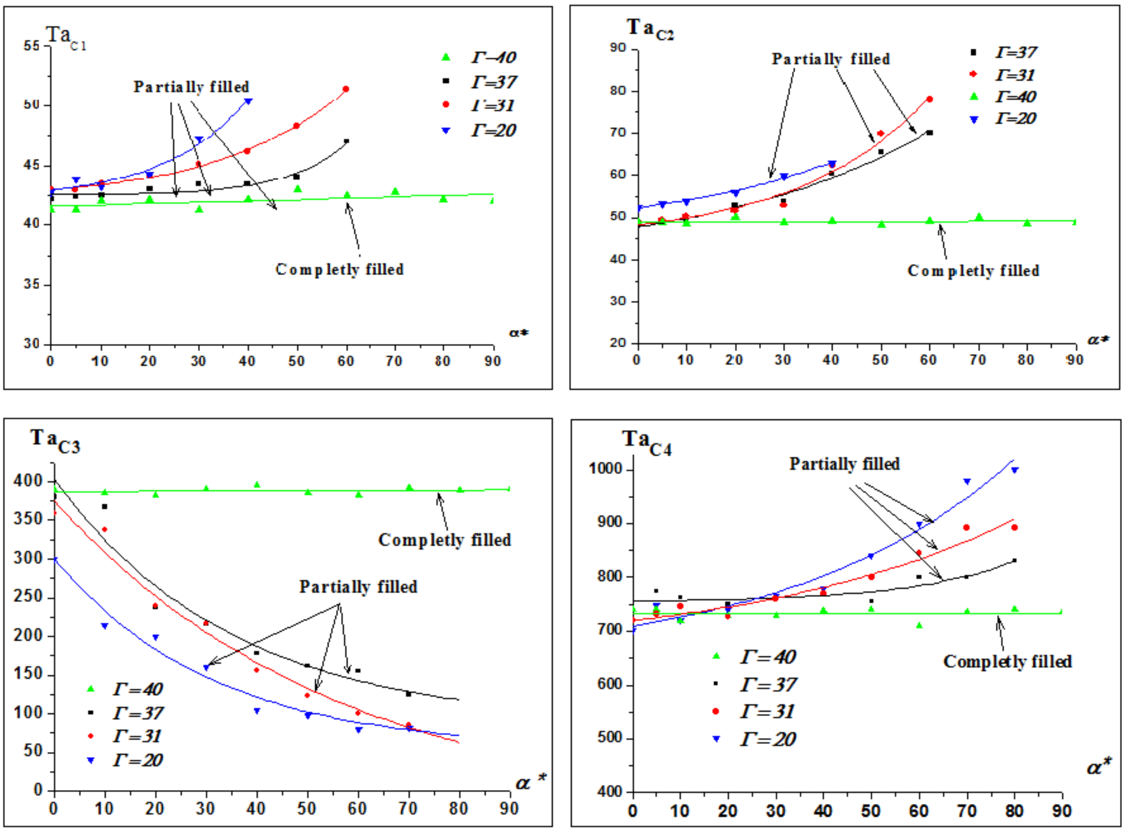


Figure 9: Evolution of critical Taylor number Tac_1, Tac_2, Tac3 and Ta_{C4} versus inclination angle α

For the system partially filled the critical Taylor numbers gradually increase for TVF, WVF and Chaos regimes if α increases. In the other hand, for the MWVF the evolution of the critical Taylor number versus inclination angle is decreased.

Table 2: Value A, B and C for different filling ratio at Tac1, Ta_{c2}, Tac3 and Ta_{c4}

Ta_{ci} / Γ	Ta_{c1}	Ta_{c2}	Ta_{c3}	Ta_{c4}
40	linear Laws *A* = 41.62±1	linear Laws *A* = 48.89±3	linear Laws *A* = 387.27±8	linear Laws *A* = 732.03±15
37	Exponential law *A* = 42.54±2 *B* = 0.025±0.003 *C* = 11.59±0.5	Exponential law *A* = 39.85±3 *B* = 8.08±0.01 *C* = 44.77±2	Exponential law *A* = 86.05±4 *B* = 317.64±9 *C* = – 34.87±1	Exponential law *A* = 754.13±20 *B* = 1.98±0.02 *C* = 21.91±0.3
31	Exponential law *A* = 42.09±2 *B* = 0.86±0.01 *C* = 25.34±0.8	Exponential law *A* = 44.61±3 *B* = 3.67±0.05 *C* = 26.85±0.3	Exponential law *A* = – 35.96±1 *B* = 411.27±8 *C* = – 56.11±3	Exponential law *A* = 660.9±20 *B* = 59.01±3 *C* = 55.73±3
20	Exponential law *A* = 42.12±2 *B* = 0.82±0.01 *C* = 17.12±1	Exponential law *A* = 45.79±3 *B* = 6.56±0.7 *C* = 41.03±2	Exponential law *A* = 53.39±3 *B* = 247.54±7 *C* = – 30.93±0.5	Exponential law *A* = 637.42±20 *B* = 71.96±3 *C* = 47.83±3

Table 3 reports the critical values of inclination angle, $\alpha^*_{C,}$ of the no-occurrence of TVF and WVF for different filling ratio. Table 3 reports the critical values of inclination angle, α^*_c, of the no-occurrence of TVF and WVF for the different filling ratio. We note that both regimes cited above have the same α^*_c for Γ=37 and 31 but not for Γ=20.

Table 3: Critical angle α^*_C versus filling ratio Γ associated to non-occurrence of TVF and WVF

Filling ratio / Flow regimes	37	31	20
TVF	70°	60°	40°
WVF	70°	60°	45°

4 Conclusion

The present experimental study made it possible to show the effect of tilted Taylor Couette flow system on the onset of different flow regimes such as TVF, WVF, MWVF, CHAOS respectively.

When the system is completely filled Γ= Γmax, the inclination angle has no effect on the flow.

However, the inclination angle has a significant influence in a system partially filled, $\Gamma<\Gamma_{max}$ giving place to various significant modifications of the movement. In addition, one revealed the effect of relaminarization of the flow for a given critical angle α_c value of the system as Γ is fixed.

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