SOLUTIONS OF NAVIER-STOKES EQUATIONS WITH NON-DIRICHLET BOUNDARY CONDITIONS IN SPACE-PERIODIC DOMAINS

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In this contribution we model flow of an incompressible fluid in a domain $\Omega^*$ which consists of subdomains $\Omega_i$ which are disjoint and moved each other in one direction and we formulate steady, non-steady and time-periodic problem. We denote by $\Omega$ one of this subdomains ($\Omega = \Omega_0$). We describe the flow by the system of the non-steady Navier-Stokes equations on a domain $\Omega$ with boundary conditions which are ”space-periodic in this direction”.

At first we describe domains $\Omega$ and $\Omega^*$. Suppose that

- $\Omega \subset \mathbb{R}^3$ is a bounded domain.
- $a \in \mathbb{R}$, $a > 0$, $\vec{\psi} = (a, 0, 0)$ is a vector.
- $\partial \Omega = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3$, $\Gamma_1$, $\Gamma_2$ and $\Gamma_3$ are open disjoint subsets of $\partial \Omega$.
- $\Gamma_2 \cap \Gamma_3 \equiv \emptyset$.
- One-dimensional measures of $\Gamma_1 \cap \Gamma_2$ and $\Gamma_2 \cap \Gamma_3$ are zero.
- $(\Omega + \vec{\psi}) \cap \Omega = \Gamma_3$ (By the symbol $A + \vec{\psi}$ we mean set of all points $[x + a, y, y] \in \mathbb{R}^3$ where $[x, y, z] \in A$.)
- $\Gamma_2 + \vec{\psi} = \Gamma_3$.

By the symbol $\Omega^*$ we denote $\bigcup_{j \in \{-\infty, \ldots, \infty\}} (j \cdot \vec{\psi} + \Omega)$.

Let $\tilde{f}$, $u_0$ be functions on $\tilde{\Omega}$ such that

$$\tilde{f}(x + a, y, z) = \tilde{f}(x, y, z)$$

and

$$\tilde{u}_0(x + a, y, z) = \tilde{u}_0(x, y, z).$$

For simplicity we denote $f = f|_{\Omega_0} = f|_{\Omega}$, $u_0 = u_0|_{\Omega_0} = u_0|_{\Omega}$, $Q = \Omega \times (0, T)$, where $(0, T)$ is a time interval, $0 < T < \infty$. We deal with the system

$$\begin{align*}
\frac{\partial u}{\partial t} + (\nabla u) u & = -\nabla p + \nu \Delta u + f \quad \text{in } Q \\
\nabla \cdot u & = 0 \quad \text{in } Q \\
\nu \frac{\partial u}{\partial n} & = 0 \quad \text{on } \partial Q \\
\frac{u \cdot n}{\sigma^{1/2}} & = 0 \quad \text{on } \partial Q \\
\end{align*}$$

with initial and boundary conditions

$$u(x, y, z, 0) = u_0(x, y, z),$$

$$\frac{u \cdot n}{\sigma^{1/2}} = 0 \quad \text{on } \partial Q.$$
\[ \frac{du}{dt} - \nu \Delta u + (u \cdot \nabla) u + \nabla P = f \quad \text{on } Q, \] (1)
\[ \text{div } u = 0 \quad \text{on } Q, \] (2)
\[ u(., 0) = \gamma \quad \text{on } \Omega, \] (3)
\[ u|_{\Gamma_1} = 0, \] (4)
\[ u|_{\Gamma_2} = u|_{\Gamma_3}, \] (5)
\[ (-P n + \nu \partial u/n)|_{\Gamma_2} = (-P n + \nu \partial u/n)|_{\Gamma_3}. \] (6)

Here \( u = (u_1, \ldots, u_m) \) denotes the velocity, \( P \) represents the pressure, \( \nu \) denotes the kinematic viscosity, \( g \) is a body force, \( \sigma \) is a prescribed vector function on \( \Gamma_2 \), \( n = (n_1, \ldots, n_m) \) is the outer normal vector on \( \partial \Omega \) and \( \gamma \) is an initial velocity. We suppose for simplicity that \( \nu = 1 \) throughout the whole paper.

Suppose that there exists a strong solution of problem (1)–(11) for given data. To prove result of existence of a strong solution for a data which are small perturbation of the previous one we use methods which is motivated by the technique described in [1]–[7].

Corresponding steady problem is the following:
\[ -\nu \Delta u + (u \cdot \nabla) u + \nabla P = g \quad \text{on } \Omega, \] (7)
\[ \text{div } u = 0 \quad \text{on } \Omega, \] (8)
\[ u|_{\Gamma_1} = 0, \] (9)
\[ u|_{\Gamma_2} = u|_{\Gamma_3}, \] (10)
\[ (-P n + \nu \partial u/n)|_{\Gamma_2} = (-P n + \nu \partial u/n)|_{\Gamma_3}. \] (11)

Here, we want to prove local solvability in the neighbourhood of famous solution also. Moreover, we want to prove regularity of corresponding Stokes solution.

We formulate also time-periodic problem on time interval \((0, T)\).:
\[ \frac{du}{dt} - \nu \Delta u + (u \cdot \nabla) u + \nabla P = f \quad \text{on } Q, \] (12)
\[ \text{div } u = 0 \quad \text{on } Q, \] (13)
\[ u(., 0) = u(., T) \quad \text{on } \Omega, \] (14)
\[ u|_{\Gamma_1} = 0, \] (15)
\[ u|_{\Gamma_2} = u|_{\Gamma_3}, \] (16)
\[ (-P n + \nu \partial u/n)|_{\Gamma_2} = (-P n + \nu \partial u/n)|_{\Gamma_3}. \] (17)

Here, we want to characterize the set of solution such that the problem is local solvable in their neighbourhood.

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Reference


