[1]

Interaction of a channel flow and moving bodies

Martin Růžička*), Miloslav Feistauer*), Jaromír Horáček) and Petr Sváček***)**
***) Charles University Prague, Faculty of Mathematics and Physics, **) Academy of Sciences of the Czech Republic, Institute of Thermomechanics, ***) Czech Technical University Prague, Faculty of Mechanical Engineering**

Introduction

The subject of this paper is the numerical simulation of the interaction of two-dimensional incompressible viscous flow through a channel (wind tunnel) and a vibrating airfoil. A solid airfoil with two degrees of freedom, which can rotate around the elastic axis and oscillate in the vertical direction, is considered. The numerical simulation consists of the finite element solution of the Navier-Stokes equations coupled with the system of ordinary differential equations describing the airfoil motion. We discuss the discretization of the problem and present some computational results.

Formulation of the problem

The two-dimensional non-stationary flow of viscous, incompressible fluid is considered in the time interval $[0, T]$, where $T > 0$. The symbol Ω_t denotes the computational domain occupied by the fluid at time t. The flow is characterized by the velocity field $\boldsymbol{u} = \boldsymbol{u}(x, t)$, and the kinematic pressure $p = p(x, t)$, for $x \in \Omega_t$ and $t \in [0, T]$. Further, $\alpha(t)$ and $h(t)$ denote the rotation angle and displacement of the airfoil.

The fluid flow is described by the Navier-Stokes system written in the ALE (Arbitrary Lagrangian–Eulerian) form (see, e.g. [1])

$$\frac{D^A}{Dt}\,\boldsymbol{u} + [(\boldsymbol{u} - \boldsymbol{w}) \cdot \nabla]\,\boldsymbol{u} + \nabla p - \nu \Delta \boldsymbol{u} = 0, \quad \mathrm{div}\,\boldsymbol{u} = 0 \quad \text{in } \Omega_t,$$

where $\nu > 0$ denotes the kinematic viscosity of the fluid, $\frac{D^A}{Dt}$ is the ALE derivative and $\boldsymbol{w}$ is the ALE velocity. (See, e.g. [1].)

The equations for the moving profile were derived from the Lagrange equations for the generalized coordinates h and α ([1]). In the matrix calculus these equations have the form

$$\widehat{\boldsymbol{K}}\boldsymbol{d}(t) + \widehat{\boldsymbol{B}}\dot{\boldsymbol{d}}(t) + \widehat{\boldsymbol{M}}\ddot{\boldsymbol{d}}(t) = \widehat{\boldsymbol{f}}(t), \tag{1}$$

where the stiffness matrix $\widehat{\boldsymbol{K}}$, the viscous damping $\widehat{\boldsymbol{B}}$ and the mass matrix $\widehat{\boldsymbol{M}}$ have the form

$$\widehat{\boldsymbol{K}} = \begin{pmatrix} k_{hh} & k_{h\alpha} \\ k_{\alpha h} & k_{\alpha\alpha} \end{pmatrix}, \quad \widehat{\boldsymbol{B}} = \begin{pmatrix} D_{hh} & D_{h\alpha} \\ D_{\alpha h} & D_{\alpha\alpha} \end{pmatrix}, \quad \widehat{\boldsymbol{M}} = \begin{pmatrix} m & S_\alpha \\ S_\alpha & I_\alpha \end{pmatrix}$$

and the vector of the force $\widehat{\boldsymbol{f}}$ and the generalized coordinates $\boldsymbol{d}$ read

$$\widehat{\boldsymbol{f}}(t) = \begin{pmatrix} -L_2(t) \\ \mathcal{M}(t) \end{pmatrix}, \quad \boldsymbol{d} = \begin{pmatrix} h(t) \\ \alpha(t) \end{pmatrix}.$$

[1]This research was supported by the grant No. IAA 200760613 of the GAASCR. The research of M. Růžička was also partly supported by the project No. 7486/2007 of the GAChU in Prague.